EES 851 Dynamic Meteorology Dr. Yuh-Lang Lin
Dept of Energy & Environ. Systems (EES) ylin@ncat.edu
Dept of Physics, NC A&T State University http://mesolab.org

4.3 Trajectories, Streamlines and Streamfunctions

(Ref.: Hess)

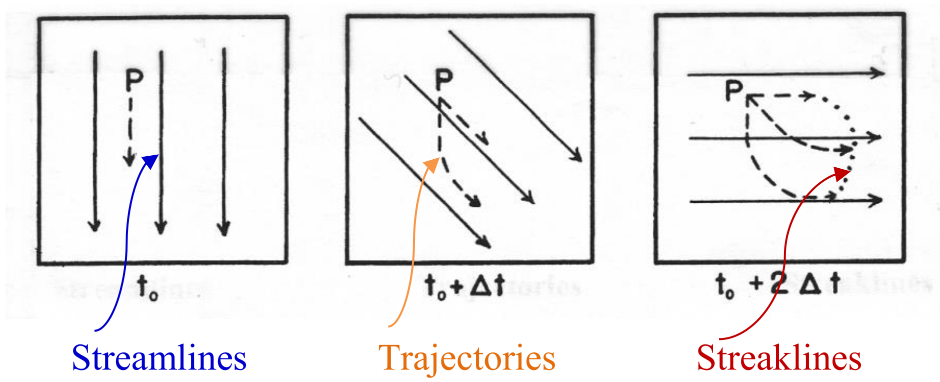
(a) Trajectory: The path followed by a particular fluid particle over a finite period of time. It is also called a path line.

- If you are interested in forward or backward trajectories of an air parcel, you can do it online from the NOAA HYSPLIT model (Homework assignment: Self practice the forward and backward trajectories for an air parcel leaving or arriving Winston-Salem at a certain time using NOAA ARL's HYSPLIT model: http://www.arl.noaa.gov => get and run HYSPLIT)

(b) Streakline: A line connecting all the particles that have passed a given geometrical point, such as a plume of smoke from a chimney.

(c) Streamline: The line which is everywhere parallel to the instantaneous flow velocity. It gives a snapshot of the velocity field at any instant, e.g. isobars are streamlines of the gradient wind in the atmosphere. Patterns of streamlines are useful in providing a pictorial representation of a flow. In a steady flow, streamlines and trajectories are identical.

(d) Streamfunction: Under certain conditions, the streamlines may be represented to a streamfunction.



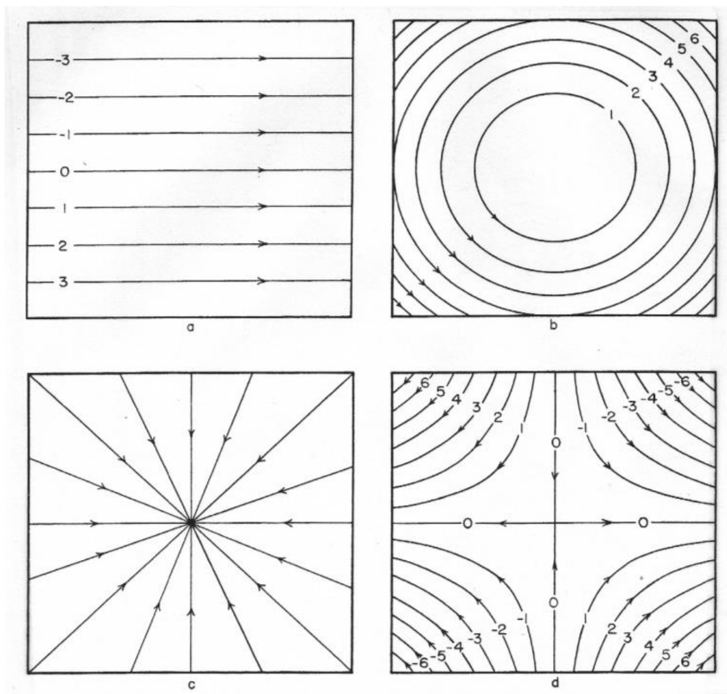


FIG. 13.3. Streamlines of pure constant—
(a) translation, (b) vorticity, (c) divergence, and (d) deformation

(Hess 1979)

Based on the definition of streamlines,

Line contours // velocity

$$\Rightarrow \quad \frac{dy}{dx} = \frac{v}{u}$$

Thus, if u and v are known functions, y can be solved to get the streamlines. Practically, u and v can be obtained from measurements, such as soundings.

Claim: For a horizontal velocity field with no divergence, such as large-scale flow and geostrophic flow, a function can be defined to label streamlines. This function is called "streamfunction."

Proof: The divergence of a non-divergent flow is

$$D = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \, .$$

We may define a function $\psi(x, y)$ such that

$$u = -\frac{\partial \psi}{\partial y} \, , \quad v = -\frac{\partial \psi}{\partial x} \, .$$

This function is called "streamfunction".

Along a streamline, the equation

$$\frac{dy}{dx} = \frac{v}{u}$$

becomes

$$\frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy = 0 \quad \text{or} \quad d\psi = 0 \, .$$

That is, ψ is constant along a streamline in a nondivergent flow. Thus, each streamline can be labeled with its value of streamfunction.

Claim: A streamfunction can be defined to label streamlines for an incompressible, two-dimensional (in x and z directions) flow.

Proof: (homework)