

## Chapter 4 Elementary Applications of the Basic Equations

### 4.5 Vertical Motion

Severe weather, such as cyclones and tornadoes, are often associated with:

- (1) *Strong upward motion* – needed for cloud formation, especially deep convection associated with cumulus clouds
- (2) *Strong cyclonic vorticity* – needed for cyclogenesis (midlatitude or tropical) and tornado genesis.

For synoptic-scale motion,  $w \sim O(1 \text{ cm/s})$

However, soundings only give an accuracy of  $O(1 \text{ m/s})$ .

Thus,  $w$  is often indirectly inferred from other measurable fields, such as  $u$ ,  $v$ ,  $T$  &  $p$ .

In order to do so, we look for equations which contain  $w$  or  $\square$ !

Two commonly used methods for inferring the vertical motion using isobaric (pressure) coordinates are:

1. **Kinematic method** – based on **continuity equation**
2. **Adiabatic method** – based on **thermodynamic energy equation**

- Relationship between  $w$  and  $\omega$

$$\omega \equiv \frac{Dp}{Dt} = \frac{\partial p}{\partial t} + \mathbf{V} \cdot \nabla p + w \frac{\partial p}{\partial z} \quad (3.36)$$

We may write  $\mathbf{V} = \mathbf{V}_g + \mathbf{V}_a$  and assume  $\mathbf{V}_a \ll \mathbf{V}_g$  for synoptic flow.

Substituting  $\mathbf{V} = \mathbf{V}_g + \mathbf{V}_a$  into (3.36) and applying the hydrostatic equation lead to

$$\omega \equiv \frac{Dp}{Dt} = \frac{\partial p}{\partial t} + \mathbf{V}_a \cdot \nabla p - g\rho w \quad (3.37)$$

Since

$$\mathbf{V}_g = \frac{1}{\rho f} \mathbf{k} \times \nabla p$$

so that  $\mathbf{V}_g \cdot \nabla p = 0$ .

Comparing the magnitudes of the three terms on the right in (3.37), we find that for synoptic-scale motions

$$\begin{aligned} \frac{\partial p}{\partial t} &\sim 10 \text{ hPa d}^{-1} \\ \mathbf{V}_a \cdot \nabla p &\sim (1 \text{ ms}^{-1})(10 \text{ hPa km}^{-1}) \sim 1 \text{ hPa d}^{-1} \\ \rho g w &\sim 100 \text{ hPa d}^{-1}. \end{aligned}$$

Thus, it is a good approximation to let

$$\omega = -g\rho w \quad (3.38)$$

for synoptic-scale motions.

### (1) The Kinematic Method

One method of deducing the vertical velocity is based on integrating the continuity equation in the vertical. Integration of (3.5)

$$\left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)_p + \frac{\partial \omega}{\partial p} = 0 \quad (3.5)$$

with respect to pressure from reference level  $p_s$  to any level  $p$  yields

$$\begin{aligned} \omega(p) &= \omega(p_s) - \int_{p_s}^p \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)_p dp. \\ &= \omega(p_s) + (p_s - p) \left( \frac{\partial \langle u \rangle}{\partial x} + \frac{\partial \langle v \rangle}{\partial y} \right)_p \end{aligned} \quad (3.39)$$

Here the angle brackets denote a pressure-weighted vertical average:

$$\langle \rangle = \frac{1}{(p_s - p)} \int_{p_s}^p ( ) dp.$$

With the aid of (3.38), the averaged form of (3.39) can be rewritten as

$$w(z) = \frac{\rho(z_s)w(z_s)}{\rho(z)} - \left( \frac{p_s - p}{\rho(z)g} \right) \left( \frac{\partial \langle u \rangle}{\partial x} + \frac{\partial \langle v \rangle}{\partial y} \right). \quad (3.40)$$

where  $z$  and  $z_s$  are the heights of pressure levels  $p$  and  $p_s$ , respectively.

The divergence term may be approximated by finite difference

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \approx \frac{u(x_o + d) - u(x_o - d)}{2d} + \frac{v(y_o + d) - v(y_o - d)}{2d}$$

**Disadvantages of the kinematic method:** Sensitive to the measured horizontal velocities.

## (2) The Adiabatic Method

$$\omega = \frac{1}{s_p} \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) \quad (3.42)$$

**Disadvantages of the adiabatic method:**

- (1)  $\partial T / \partial t$  is required and the time interval is too large for synoptic observations,
- (2) Only works at where far away from PBL or strong convection.