EES 851 Dynamic Meteorology
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Chapter 7 Quasi-Geostrophic (QG) Theory for Midlatitude Synoptic Systems

7.3 Geopotential Tendency Equation

(Equation editor: $D/Dt = \partial/\partial t + u\partial/\partial x$)

Purpose: To derive a prognostic equation for predicting geopotential tendency.

- Based on the hydrostatic equation

$$\frac{\partial \phi}{\partial p} = -\frac{RT}{p} \qquad (6.2)$$

We have

$$T = -\frac{p}{R}\frac{\partial \phi}{\partial p}$$

Substituting it into the QG thermodynamic equation

$$\frac{\partial T}{\partial t} = -u_g \frac{\partial T}{\partial x} - v_g \frac{\partial T}{\partial y} + \left(\frac{\sigma p}{R}\right)\omega + \frac{J}{c_p} \qquad (6.13)$$

leads to

$$\frac{\partial \chi}{\partial p} = -V_g . \nabla \frac{\partial \phi}{\partial p} - \sigma\omega - \frac{\kappa J}{p} \qquad (6.22)$$

where $\kappa = R/c_p$.

- Equation (6.22) is also called "hydrostatic thermodynamic equation".

  Q: What is the physical meaning of individual terms of (6.22)?

- Equations (6.22) and the QG vorticity equation (derived in Ch.7.2)

$$\frac{\partial \zeta_g}{\partial t} + u_g \frac{\partial \zeta_g}{\partial x} + v_g \frac{\partial \zeta_g}{\partial y} = f_0 \frac{\partial \omega}{\partial p} - \beta v_g \qquad (6.18)$$

or

$$\frac{1}{f_0}\nabla^2 \chi + u_g \frac{\partial}{\partial x}\left(\frac{1}{f_0}\nabla^2 \phi\right) + v_g \frac{\partial}{\partial y}\left(\frac{1}{f_0}\nabla^2 \phi\right) = f_0 \frac{\partial \omega}{\partial p} - \beta v_g \qquad (6.18)'$$

form a closed set of equations since

$$\zeta_g = \frac{1}{f_0}\nabla_p^2 \phi \qquad (6.13)$$

- Eliminate ω ⇨ geopotential tendency (χ) equation
  ⇨ To predict geopotential height tendency
- Eliminate χ ⇨ Omega (ω) equation
  ⇨ To diagnose vertical motion

The geopotential tendency equation can then be derived

$$\left[\nabla^2 + \frac{\partial}{\partial p}\left(\frac{f_o^2}{\sigma}\frac{\partial}{\partial p}\right)\right]\chi = -f_o V_g.\nabla(\frac{1}{f_o}\nabla^2\phi + f) - \frac{\partial}{\partial p}\left[\frac{-f_o^2}{\sigma}V_g \bullet \nabla\left(-\frac{\partial\phi}{\partial p}\right)\right] - (f_o^2\kappa)\frac{\partial}{\partial p}\left(\frac{J}{\sigma}\right)$$

Term A Term B Term C Term D

(6.23)

Physical meaning of (6.23) may be understood by the following simple form:

$$-\chi \propto -V_{\mathbf{g}}\cdot\nabla\left(\zeta_g + f\right) + \frac{\partial}{\partial z}\left(-V_{\mathbf{g}}\cdot\nabla T\right)$$

or

$$-\chi \propto -V_{\mathbf{g}}\cdot\nabla\zeta_g - \beta v_g + \frac{\partial}{\partial z}\left(-V_{\mathbf{g}}\cdot\nabla T\right)$$

Term B: Relative and Planetary Vorticity Advection

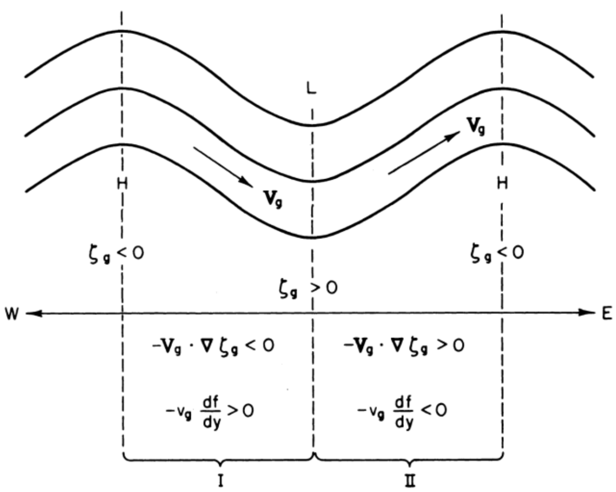
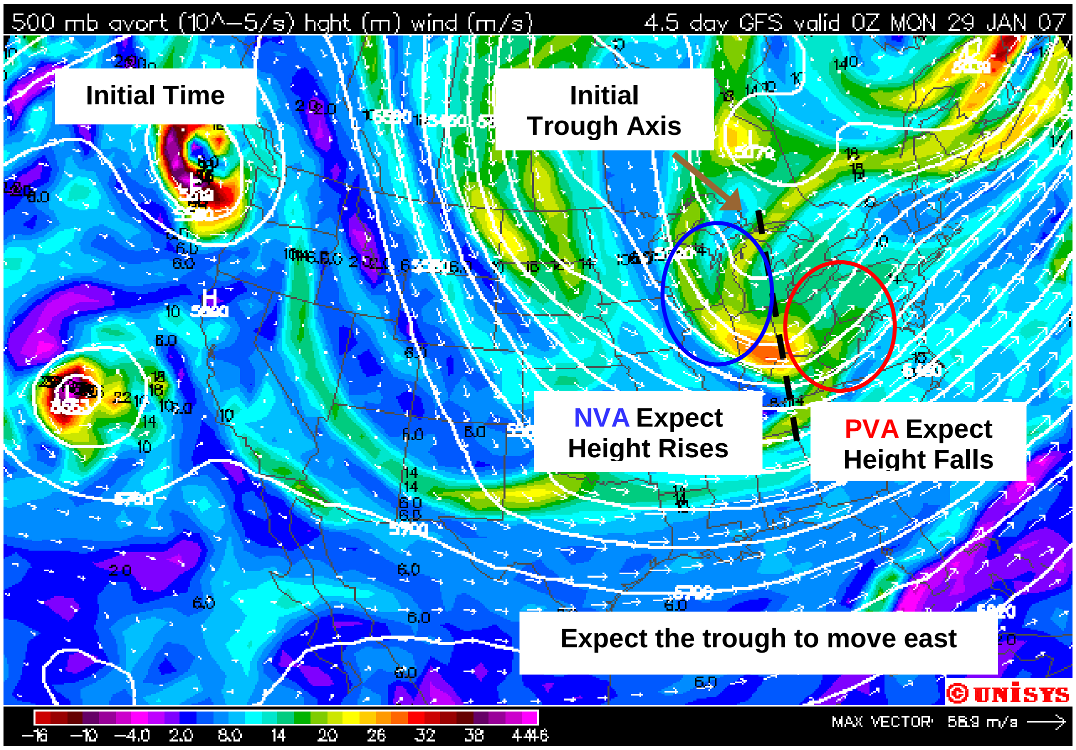
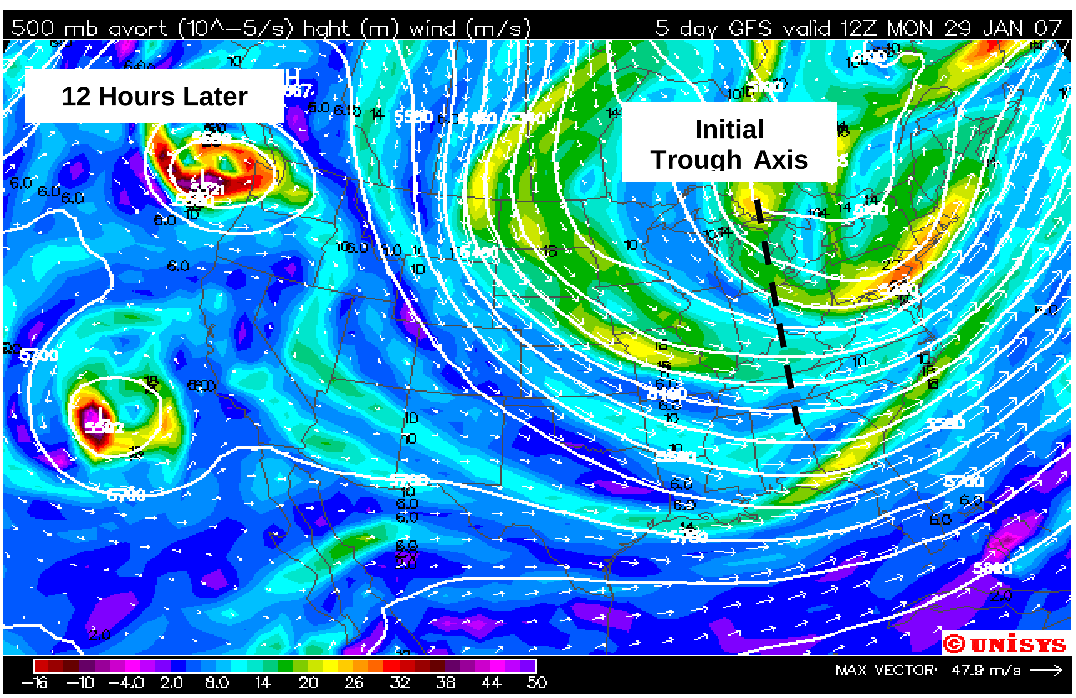


Fig. 6.7 Schematic 500-hPa geopotential field showing regions of positive and negative advections of relative and planetary vorticity.

In Region I: ϕ increases (decreases) with time due to relative (planetary) vorticity advection.

Example of Relative Vorticity Advection in the Real World





Term C: Differential Temperature Advection $\left(\frac{\partial}{\partial z}\left(-V_g \cdot \nabla T\right)\right)$

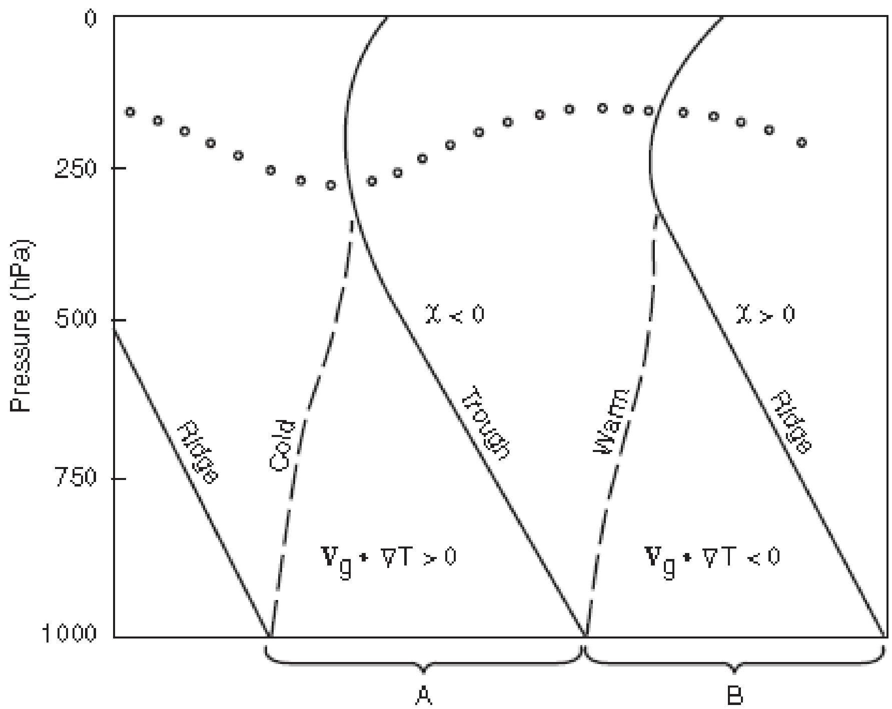


Fig. 6.9 East–west section through a developing synoptic disturbance showing the relationship of temperature advection to the upper level height tendencies. A and B designate, respectively, regions of cold advection and warm advection in the lower troposphere.

In Region A (B): ϕ decreases (increases) with time due to cold (warm) advection.

Example of Warm Advection in the Real World