7.4 QG Diagnosis: Vertical Motion

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Diagnoze vertical motion in the atmosphere:

Our Challenge:

- We do not observe vertical motion
- Intimately linked to clouds and precipitation
- Actual vertical motions are often several orders of magnitude smaller than their collocated horizontal air motions [w ~ 0.01 → 10 m/s]
  [u,v ~ 10 → 100 m/s]
- Synoptic-scale vertical motions must be estimated from widely-spaced observations (i.e. the rawindsonde network) every 12-hours

Methods:

- Kinematic Method — Integrate the Continuity Equation
  Very sensitive to small errors in winds measurements

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial \omega}{\partial p}=0,$$

to estimate ω at p,

$$\omega(p)=\omega(p_s)+(p_s-p)\left[\frac{\partial\langle u\rangle}{\partial x}+\frac{\partial\langle v\rangle}{\partial y}\right]_p . \qquad \text{H(3.38)}$$

Adapted from Dr. Eastin's Lecture Note (UNCC)

QG Diagnosis: Vertical Motion

- Adiabatic Method — From the thermodynamic equation
  Very sensitive to temperature tendencies (too coarse)
  Difficult to incorporate impacts of diabatic heating

$$\omega = \frac{1}{S_p}\left[\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right]. \qquad \text{H(3.41)}$$

- QG Omega Equation — Least sensitive to small observational errors
  Widely believed to be the best method

QG Prediction: System Evolution

The Quasigeostrophic Height-Tendency Equation:

- We can now derive a ***single*** prognostic equation for ϕ (diagnostic equation for χ) by combining our new vorticity and thermodynamic equations:

$$\frac{1}{f_0}\nabla_p^2\chi + u_g\frac{\partial}{\partial x}\left(\frac{1}{f_0}\nabla_p^2\phi\right) + v_g\frac{\partial}{\partial y}\left(\frac{1}{f_0}\nabla_p^2\phi\right) = f_0\frac{\partial\omega}{\partial p} - \beta v_g$$

$$\frac{\partial\chi}{\partial p} + u_g\frac{\partial}{\partial x}\left(\frac{\partial\phi}{\partial p}\right) + v_g\frac{\partial}{\partial y}\left(\frac{\partial\phi}{\partial p}\right) = -\sigma\omega$$

- To do this, we need to eliminate the vertical motion (ω) from both equations

Step 1: Apply the operator $\frac{f_0}{\sigma}\frac{\partial}{\partial p}$ to the thermodynamic equation

Step 2: Add the result of Step 1 to the vorticity equation

After some math, we get the resulting diagnostic equation……

QG Diagnosis: Vertical Motion

The Quasigeostrophic Omega Equation:

- We can also derive a ***single*** diagnostic equation for ω by, again, combining our vorticity and thermodynamic equations (the height-tendency versions from before):

$$\frac{1}{f_0}\nabla_p^2\chi + u_g\frac{\partial}{\partial x}\left(\frac{1}{f_0}\nabla_p^2\phi\right) + v_g\frac{\partial}{\partial y}\left(\frac{1}{f_0}\nabla_p^2\phi\right) = f_0\frac{\partial\omega}{\partial p} - \beta v_g$$

$$\frac{\partial\chi}{\partial p} + u_g\frac{\partial}{\partial x}\left(\frac{\partial\phi}{\partial p}\right) + v_g\frac{\partial}{\partial y}\left(\frac{\partial\phi}{\partial p}\right) = -\sigma\omega$$

- To do this, we need to eliminate the height tendency (χ) from both equations

Step 1: Apply the operator $f_0\frac{\partial}{\partial p}$ to the vorticity equation

Step 2: Apply the operator ∇_p^2 to the thermodynamic equation

Step 3: Subtract the result of Step 1 from the result of Step 2

After some math, we get the resulting diagnostic equation.

QG Diagnosis: Vertical Motion

The BASIC Quasigeostrophic Omega Equation:

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{-\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[-\mathbf{V_g}\cdot\nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} \underbrace{-\frac{R}{\sigma p}\nabla_p^2\left(-\mathbf{V_g}\cdot\nabla_p T\right)}_{\text{Term C}}$$

Term A **Term B** **Term C**

- To obtain an ***actual value*** for ω (the ideal goal), we would need to compute the forcing terms (Terms B and C) from the three-dimensional wind and temperature fields, and then invert the operator in Term A using appropriate boundary conditions
- Again, this is not a simple task (*forecasters never do this*)…..

- Rather, we can ***infer the sign and relative magnitude*** of ω through simple inspection of the three-dimensional absolute vorticity and temperature fields (*forecasters do this all the time…*)
- Thus, let's examine the physical interpretation of each term….

Note: Again, this is ***not*** the same form of the QG omega equation shown in Holton
While equivalent to "Holton's version", this version is easier to physically understand and is the version used in our Bluestein text

QG Diagnosis: Vertical Motion

The BASIC Quasigeostrophic Omega Equation:

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{-\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[-V_{\mathbf{g}}\cdot\nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} \underbrace{- \frac{R}{\sigma p}\nabla_p^2\left(-V_{\mathbf{g}}\cdot\nabla_p T\right)}_{\text{Term C}}$$

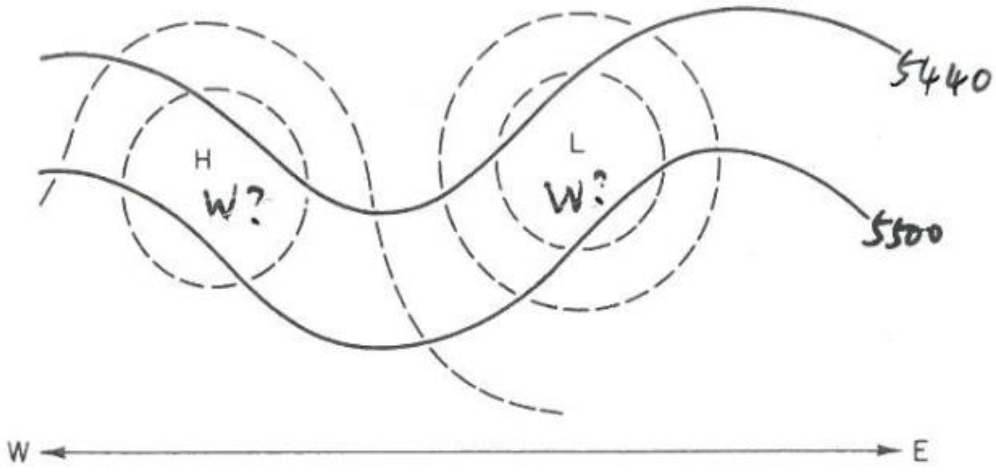
Term A → Term B → Term C →

$$w = \frac{\partial}{\partial z}\left[-V_{\mathbf{g}}\cdot\nabla_p\left(\zeta_g + f\right)\right] - V_{\mathbf{g}}\cdot\nabla_p T$$

Term A: Local Vertical Motion

- Again, if we incorporate the negative sign into our physical interpretation, which we will do, we can just think of this term as the vertical motion
- Thus, this term is **our goal – a qualitative estimate of the deep –layer synoptic-scale vertical motion at a particular location**

Effects of Term B



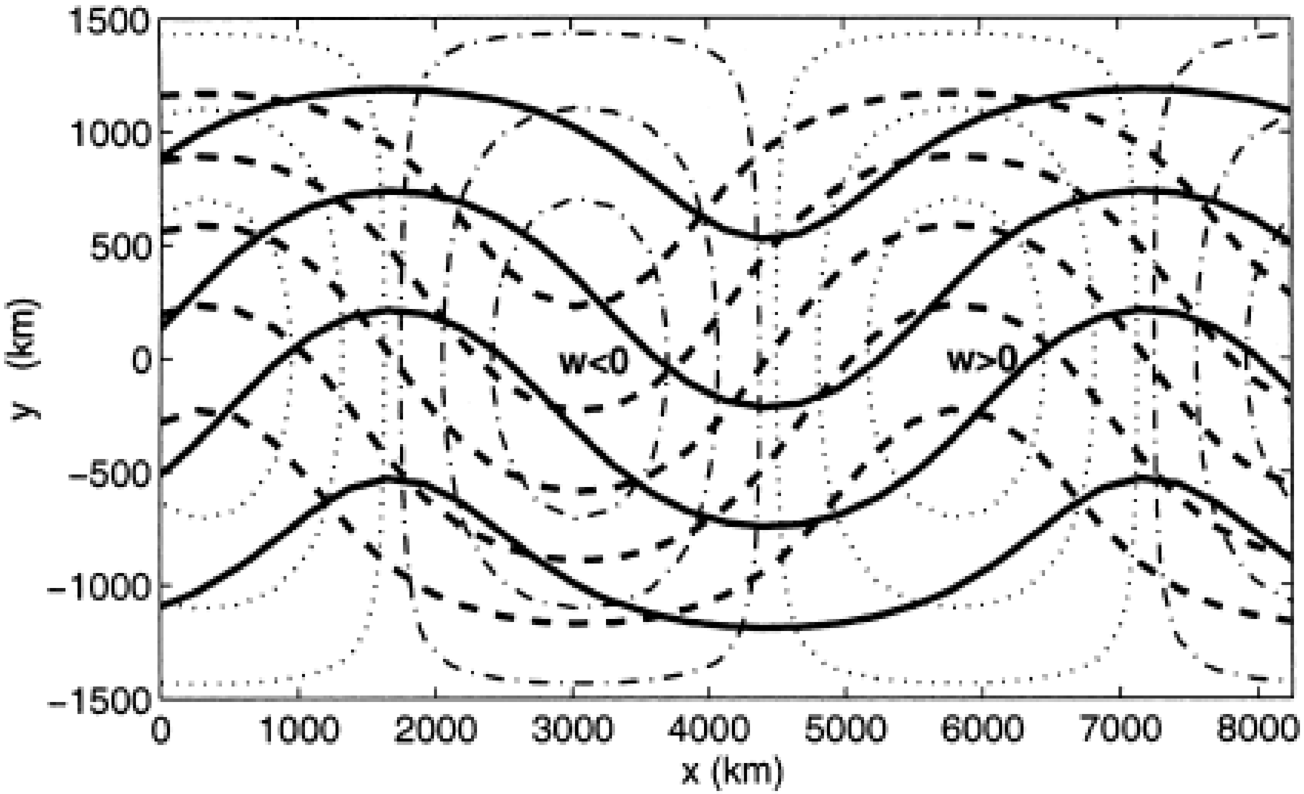


Fig. 6.12 Schematic 500-hPa height contours (solid lines), isotherms (dashed lines), and vertical motion field ($w > 0$ dash-dot lines, $w < 0$ dotted lines) for a developing synoptic-scale system. Upward motion occurs where vorticity decreases moving left to right along an isotherm, and downward motion occurs where vorticity decreases moving left to right along an isotherm.

QG Diagnosis: Vertical Motion

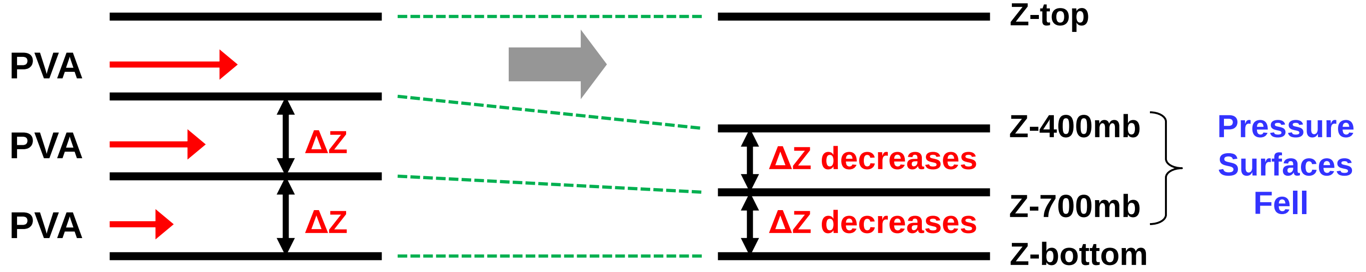
Further Explanation of The QG Omega Equation (Term B):

$$w \;=\; \frac{\partial}{\partial Z}\left[-V_{\mathbf{g}} \cdot \nabla_p \left(\zeta_g + f\right)\right] \quad -V_{\mathbf{g}} \cdot \nabla_p T$$

Term A **Term B** **Term C**

Term B: Change in Absolute Vorticity Advection with "Height"

- Recall, positive (relative) vorticity advection (**PVA**) leads to local height falls
- Consider a three-layer atmosphere where cyclonic vorticity advection increases with height, or **PVA** is strongest in the upper layer:



- Hydrostatic balance (and the hypsometric equation) requires ALL changes in thickness (ΔZ) to be accompanied by temperature changes…

QG Diagnosis: Vertical Motion

Further Explanation of The QG Omega Equation (Term B):

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{-\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[-\mathbf{v_g}\bullet\nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{-\frac{R}{\sigma p}\nabla_p^2\left(-\mathbf{v_g}\bullet\nabla_p T\right)}_{\text{Term C}}$$

Term B: Change in Absolute Vorticity Advection with “Height”

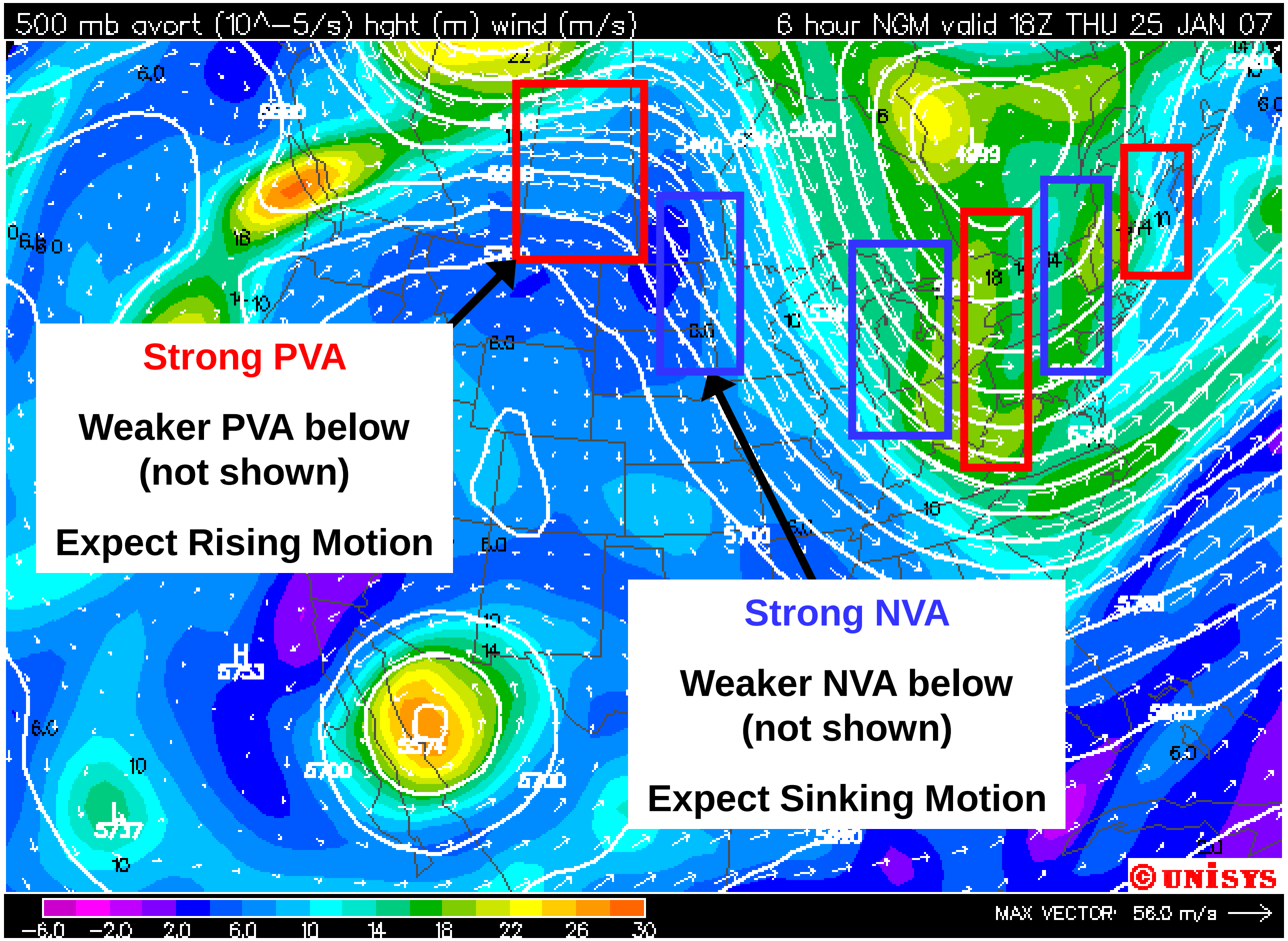
- These thickness decreases (height falls) were ***not*** a result of temperature changes
- Thus, in order to maintain hydrostatic balance, the thickness decreases must be accompanied by a temperature decrease

- In the absence of temperature advection and diabatic cooling, only adiabatic cooling associated with rising motion can create this required temperature decrease

- Therefore, an **increase in PVA with height** will induce **rising motion**

Example of the QG Omega Equation in the Real World (Term B):

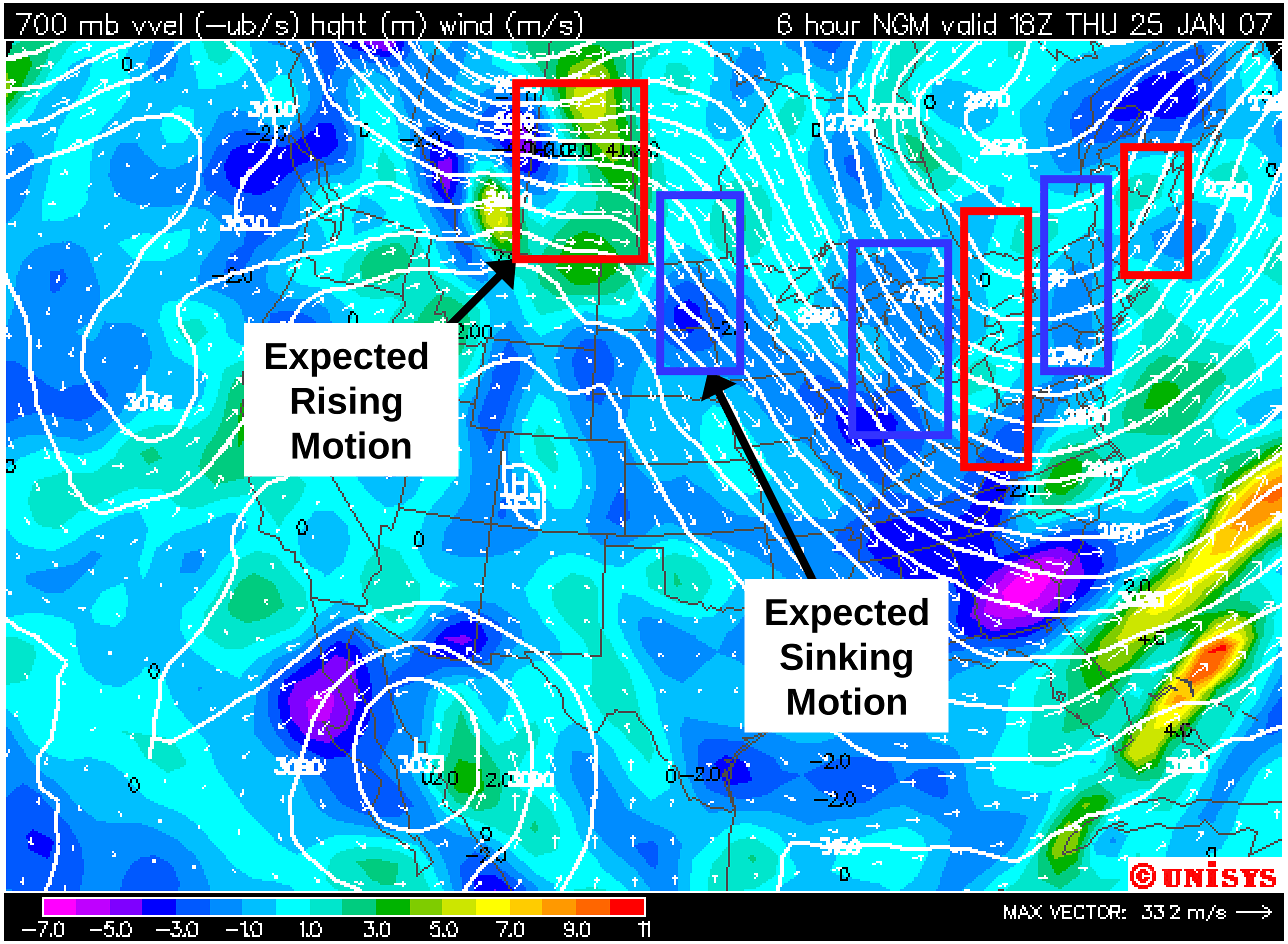
Term B: Change in Relative Vorticity Advection with “Height”



QG Diagnosis: Vertical Motion

The BASIC Quasigeostrophic Omega Equation:

Term B: Change in Absolute Vorticity Advection with "Height"

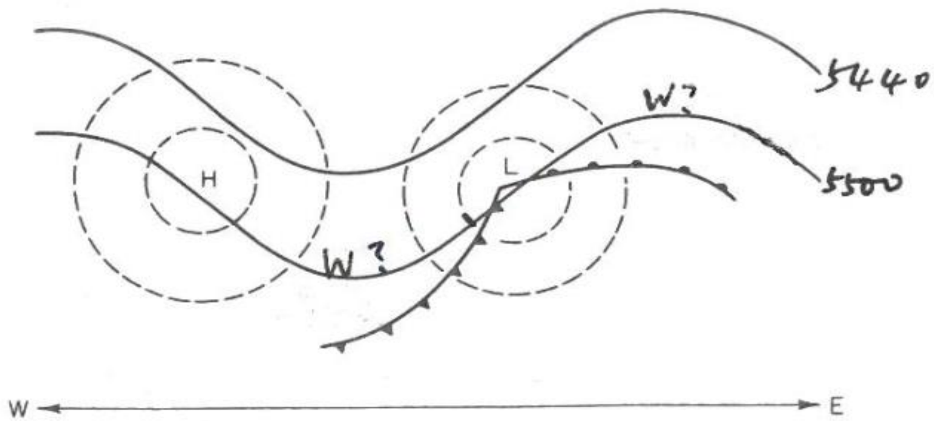


Effects of The QG Omega Equation (Term C):

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{-\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[-\mathbf{V_g}\cdot\nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} \underbrace{- \frac{R}{\sigma p}\nabla_p^2\left(-\mathbf{V_g}\cdot\nabla_p T\right)}_{\text{Term C}}$$

Term A, Term B, Term C

$$w = \frac{\partial}{\partial z}\left[-\mathbf{V_g}\cdot\nabla_p\left(\zeta_g + f\right)\right] - \mathbf{V_g}\cdot\nabla_p T$$

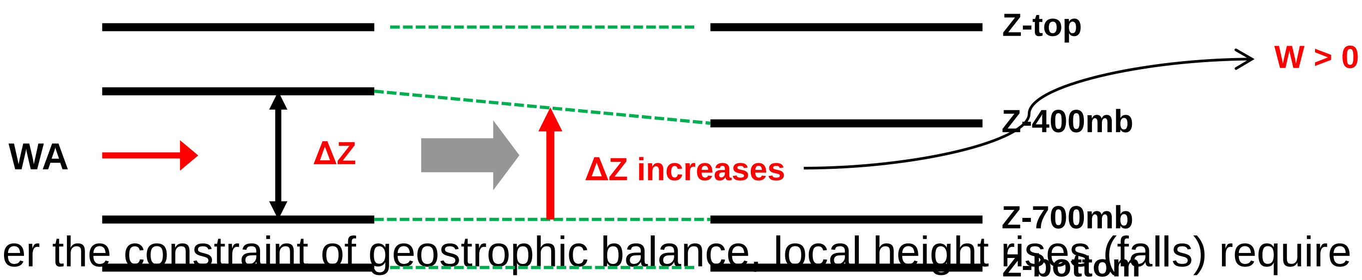


Further Explanation of The QG Omega Equation (Term C):

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{-\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[-\mathbf{v_g}\bullet\nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{-\frac{R}{\sigma p}\nabla_p^2\left(-\mathbf{v_g}\bullet\nabla_p T\right)}_{\text{Term C}}$$

Term C: Horizontal Temperature Advection

- Warm air advection (**WA**) leads to local temperature increases
- Consider the three-layer model, with **WA** strongest in the middle layer



- Under the constraint of geostrophic balance, local height rises (falls) require a change in the local pressure gradient, a change in the local geostrophic wind, and thus a local decrease (increase) in <u>geostrophic</u> vorticity…..

QG Diagnosis: Vertical Motion

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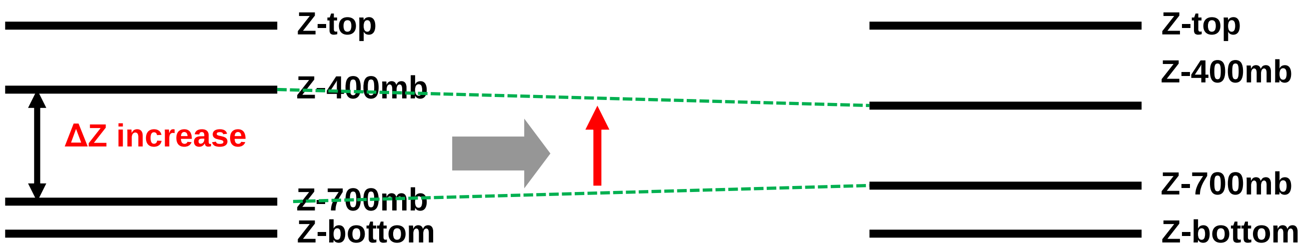
Term C: Horizontal Temperature Advection

- These height changes where ***not*** a result of changes in geostrophic vorticity
- Thus, in order to maintain geostrophic balance in the absence of vorticity advection, local height rises must be accompanied by divergence (which decreases vorticity) and height falls must be accompanied by convergence (which increases vorticity)
- Mass continuity then requires rising motion through layer

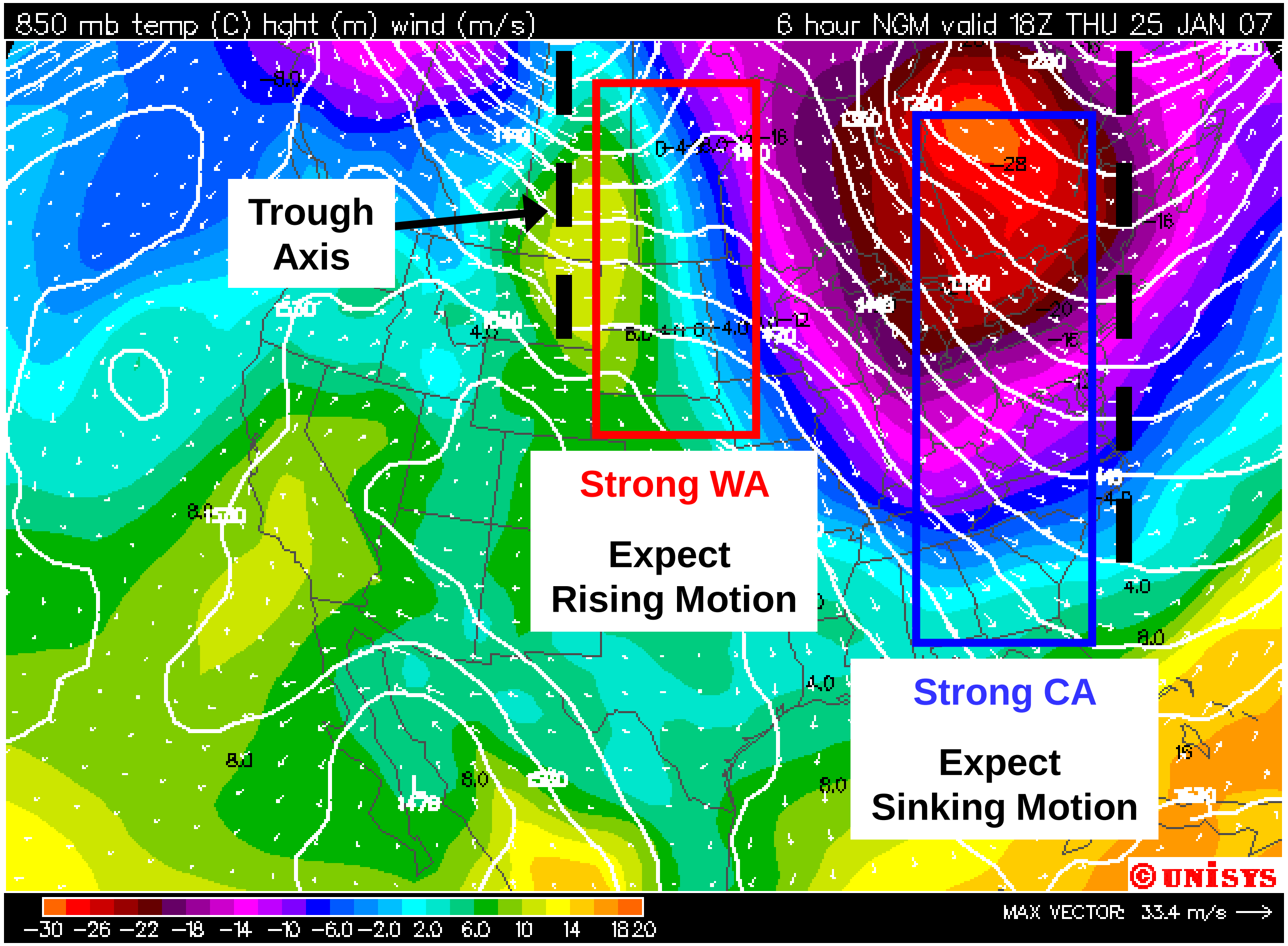
- Therefore, **WA** will induce **rising motion**



QG Diagnosis: Vertical Motion

The BASIC Quasigeostrophic Omega Equation:

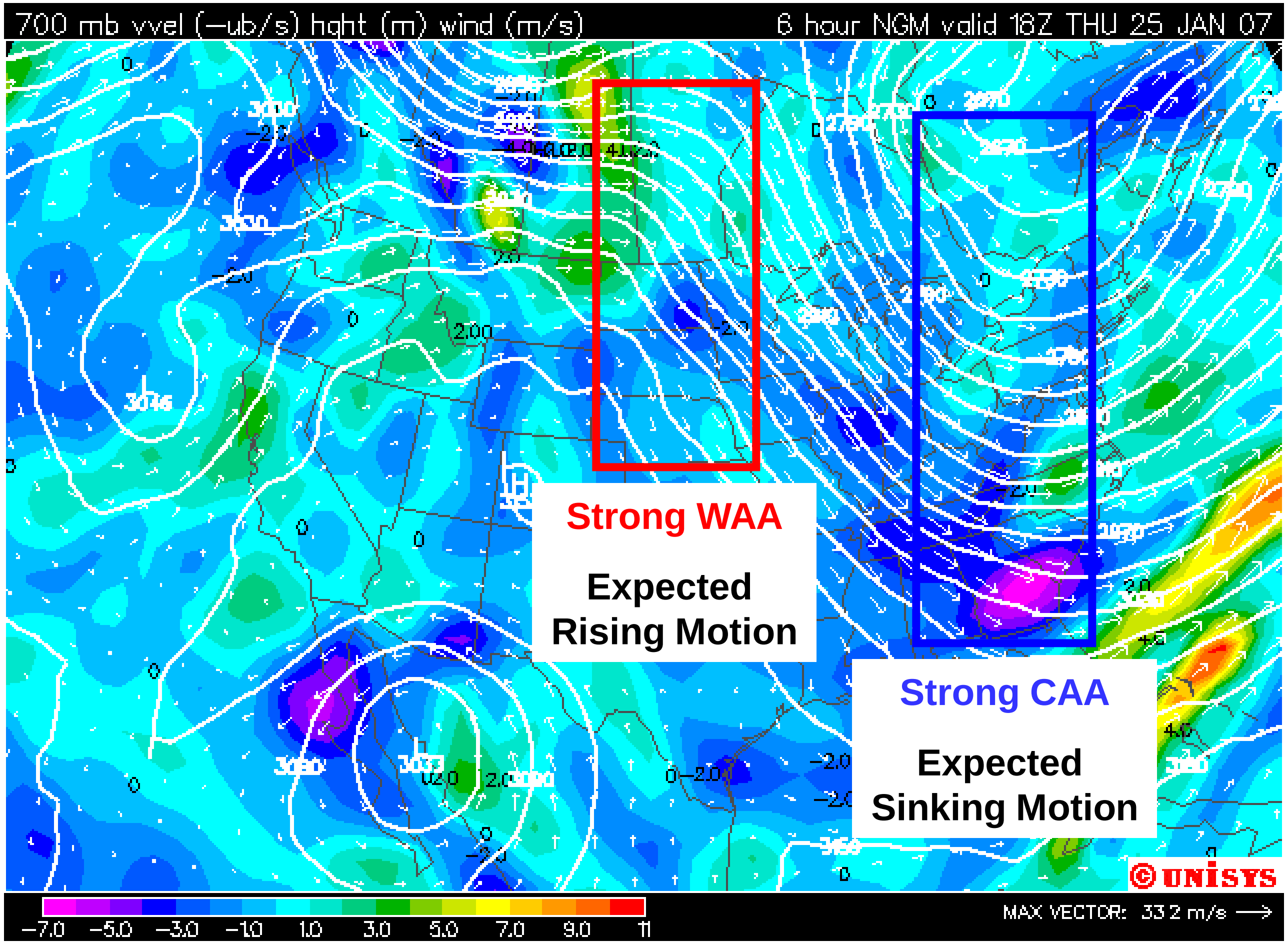
Term C: Horizontal Temperature Advection



QG Diagnosis: Vertical Motion

The BASIC Quasigeostrophic Omega Equation:

Term C: Horizontal Temperature Advection



QG Diagnosis: Vertical Motion

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Summary and Application Tips:

- You must consider the effects of both **Term B** and **Term C** at multiple levels
- If large (small) changes in the vorticity advection with height are observed, then you should expect large (small) vertical motions
- The stronger the temperature advection, the stronger the vertical motion
- If WAA (CAA) is observed at several consecutive pressure levels, expect a deep layer of rising (sinking) motion
- Opposing expectations in vertical motion from the two terms at a given location will alter the total vertical motion pattern

QG Diagnosis: Vertical Motion

Summary and Final Comments:

- The QG omega equation is a **diagnostic** equation:
  - The equation does ***not predict*** future vertical motion patterns
  - The forcing functions (Terms B and C) do not ***cause*** the expected responses, with an implied time lag between the forcing and the response
  - The responses are *instantaneous*
  - The responses are a direct result of the atmosphere maintaining hydrostatic and geostrophic balance at the time of the forcing

- Use of the QG omega equation in a **diagnostic** setting (forecasting):
  - Diagnose the **synoptic–scale** vertical motion pattern, and assume rising motion corresponds to clouds and precipitation when ample moisture is available
  - Compare to the observed patterns → Infer mesoscale contributions

- Use of the QG omega equation in a **limited prognostic** setting (forecasting):
  - Diagnose the **synoptic-scale contribution** to the total vertical motion, cloud, and precipitation patterns predicted at a future time by a numerical model
  - Help distinguish between regions of persistent precipitation (synoptic scale) and more sporadic precipitation (mesoscale)

References

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