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Chapter 4 Elementary Applications of the Basic Equations

4.1 Basic Equations in Isobaric Coordinates

(Ref.: Holton Sec. 3.1)
(Classical equation editor: $D/Dt = \partial/\partial t + u\partial/\partial x$)

- **The Horizontal Momentum Equation**

The approximate horizontal momentum equations (2.24) and (2.25) may be written in vectorial form as

$$\frac{D\boldsymbol{V}}{Dt} + f\,\boldsymbol{k}\,x\boldsymbol{V} = -\frac{1}{\rho}\nabla p \qquad (3.1)$$

where $\boldsymbol{V} = u\,\boldsymbol{i} + v\,\boldsymbol{j}$ is the horizontal velocity vector.

Substituting the geostrophic wind equations, Eqs. (1.20) and (1.21)

$$-\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)_z = -\left(\frac{\partial \phi}{\partial x}\right)_p, \qquad (1.20)$$

$$-\frac{1}{\rho}\left(\frac{\partial p}{\partial y}\right)_z = -\left(\frac{\partial \phi}{\partial y}\right)_p \qquad (1.20)$$

into (3.1) leads to

$$\frac{D\boldsymbol{V}}{Dt} + f\,\boldsymbol{k}\,x\boldsymbol{V} = -\nabla_p \phi \qquad (3.2)$$

where ∇_p is the horizontal gradient operator applied with pressure held constant.

Because *p* is the independent vertical coordinate, we must expand the total derivative as

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \frac{Dx}{Dt}\frac{\partial}{\partial x} + \frac{Dy}{Dt}\frac{\partial}{\partial y} + \frac{Dp}{Dt}\frac{\partial}{\partial p} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + \omega\frac{\partial}{\partial p}$$

(3.3)

Here $\omega = Dp/Dt$ is the "omega vertical motion," which is defined as the pressure change following the motion, similar to $w = Dz/Dt$.

For synoptic motions, $\omega \approx -\rho g w$.

- From (3.2), the geostrophic relationship can be written as

$$f\,\boldsymbol{V_g} = \boldsymbol{k}\,x\nabla_p \phi \qquad (3.4)$$

or in scalar form

$$fu_g = -\frac{\partial \phi}{\partial y}\,, \qquad (3.4a)$$

$$f v_g = \frac{\partial \phi}{\partial x}.$$

(3.4b)

Note there is no density present in (3.4)

In addition, on an *f*-plane (i.e., *f* is constant), we have

$$\nabla_p \cdot \boldsymbol{V_g} = 0$$

That is, there is no divergence for the geostrophic flow (non-divergent).

- The Continuity Equation in the isobaric coordinates becomes

$$\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)_p + \frac{\partial \omega}{\partial p} = 0 \quad . \qquad (3.5)$$

- The Thermodynamic Energy Equation

$$c_p \frac{DT}{Dt} - \alpha \frac{Dp}{Dt} = J \qquad (2.42)$$

then becomes

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} - S_p \omega = \frac{J}{c_p} \qquad (3.6)$$

where $J = \frac{Dq}{Dt}$ is the diabatic heating rate and

$$S_p \equiv \frac{RT}{c_p p} - \frac{\partial T}{\partial p} = -\frac{T}{\theta}\frac{\partial \theta}{\partial p}. \qquad (3.7)$$

or

$$S_p \equiv \frac{\Gamma_d - \Gamma}{\rho g}$$

where S_p is the "static stability parameter".