

Atmospheric Thermodynamics
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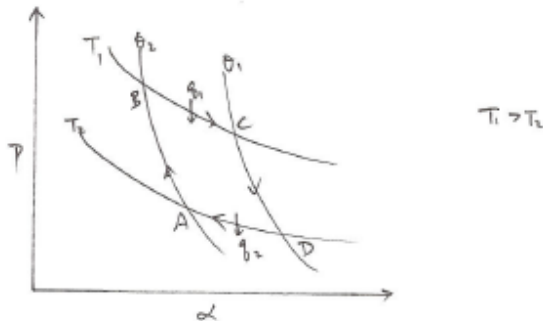
Thermodynamic Diagrams

(Based on Ch.5 of Hess)

[For the classical equation editor: ($dq = 0$)]

a. Introduction

Remember, in a previous lecture, we have represented some atmospheric processes in a α - p diagram:



(What is this? What are the processes represented in the figure?)

We also have identified some atmospheric states and processes on a (T, p^κ) diagram:

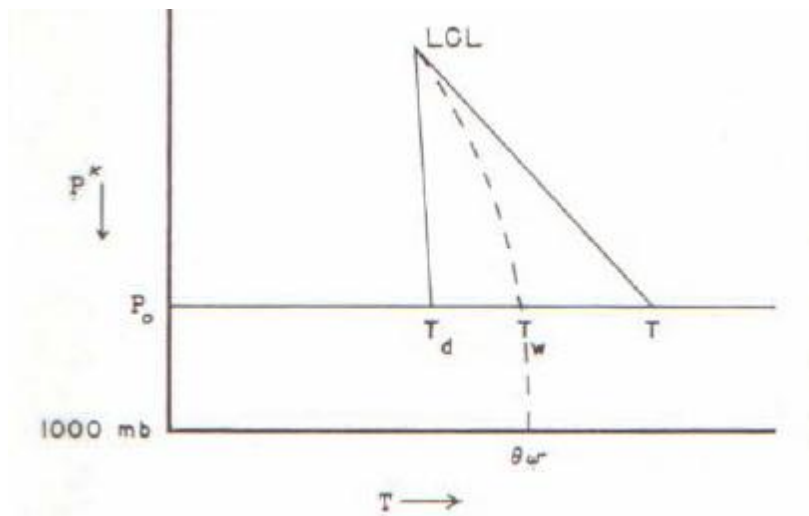


Figure 20.1 (Fig. 4.7 of Hess) ($\kappa = R_d/c_p$)

- (i) **Isobar**: the straight horizontal line.
- (ii) **Isotherm**: the straight vertical line.
- (iii) **Dry adiabat**: the straight line passing through T .
- (iv) **Constant mixing ratio line**: the straight line passing through T_d (dew point).
- (v) **Pseudoadiabat (moist adiabat)**: the dashed line passing through T_w .

This type of diagram is called “**Thermodynamic Diagram**” which helps us represent atmospheric states and processes on a chart.

These type of thermodynamic diagrams can be improved to serve the following.

- (1) Purpose: A thermodynamic diagram is to provide a graphic display of the lines representing major processes to which air may be subject, namely isobaric, isothermal, dry adiabatic, and pseudoadiabatic processes.
- (2) Fundamental lines:
 - a) isobars,
 - b) isotherms,
 - c) dry adiabats,
 - d) pseudoadiabats,
 - e) saturation mixing ratio.
- (3) Data to be represented: the data to be represented are obtained from soundings and consist of temperature, pressure, and humidity.
- (4) Desired characteristics when designed a thermodynamic diagram:
 - a) Area is proportional to work or energy;
 - b) As many as possible of the fundamental lines are straight;
 - c) As large as possible of the angle between the isotherms and the dry adiabats.

b. Equal area transformation

Consider the following two thermodynamic diagrams:

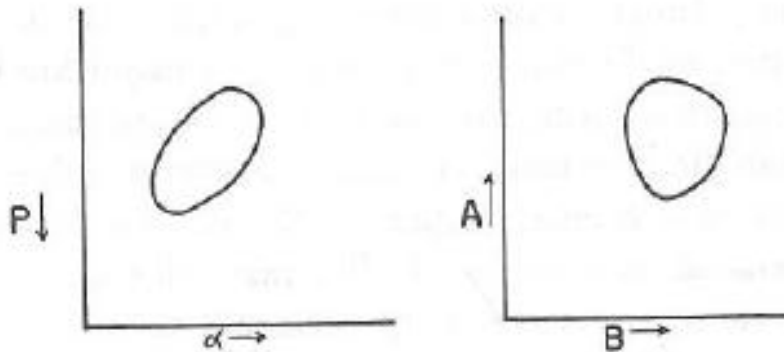


Fig. 21.1: Representation of a cycle on a α , $-p$ diagram and its equal-area transformation to an A, B diagram.

We know in a α - p diagram, the amount of work done for a cyclic process is

$$w = -\oint p d\alpha ,$$

which is the area enclosed by the closed curve in the α - p diagram.

An equal area transformation from α - p coordinates to, say, B-A coordinates requires:

$$w = -\oint p d\alpha = \oint A dB ,$$

or

$$0 = \oint pd\alpha + \oint AdB = \oint (pd\alpha + AdB). \quad (21.1.1)$$

Equation (21.1.1) implies that $pd\alpha + AdB$ is an exact differential.
 Why?

Thus, we may define a variable ϕ such that

$$d\phi = pd\alpha + AdB.$$

In other words, ϕ is a function of α and B .

$$\phi = \phi(\alpha, B)$$

or

$$d\phi(\alpha, B) = \left[\frac{\partial \phi}{\partial \alpha} \right]_B d\alpha + \left[\frac{\partial \phi}{\partial B} \right]_\alpha dB.$$

Therefore, the sufficient conditions for an equal area transformation are:

$$p = \left[\frac{\partial \phi}{\partial \alpha} \right]_B; \quad A = \left[\frac{\partial \phi}{\partial B} \right]_\alpha.$$

This implies,

$$\frac{\partial p}{\partial B} = \frac{\partial^2 \phi}{\partial \alpha \partial B}; \quad \frac{\partial A}{\partial \alpha} = \frac{\partial^2 \phi}{\partial \alpha \partial B}.$$

Therefore, if

$$\left[\frac{\partial A}{\partial \alpha} \right]_B = \left[\frac{\partial p}{\partial B} \right]_\alpha, \quad (21.1.2)$$

then the area will be equal on the two diagrams.

c. The Emagram

Consider the case where $B = T$. Then, using (21.1.2), we have

$$\left[\frac{\partial A}{\partial \alpha} \right]_T = \left[\frac{\partial p}{\partial T} \right]_\alpha.$$

Using the equation of state

$$p\alpha = RT,$$

we get,

$$\left[\frac{\partial p}{\partial T} \right]_{\alpha} = \frac{R}{\alpha}.$$

Therefore,

$$\left[\frac{\partial A}{\partial \alpha} \right]_T = \left[\frac{\partial p}{\partial T} \right]_{\alpha} = \frac{R}{\alpha},$$

or

$$\left[\frac{\partial A}{\partial \alpha} \right]_T d\alpha = \frac{R}{\alpha} d\alpha.$$

Integrate this equation, we get

$$A = R \ln \alpha + f(T)$$

where $f(T)$ can be chosen as any function of T .

Take the log of the equation of state, we find

$$\ln \alpha = -\ln p + \ln R + \ln T$$

Thus,

$$A = -R \ln p + [R \ln R + R \ln T + f(T)]$$

Therefore, if we choose $f(T) = -R \ln R - R \ln T$, then

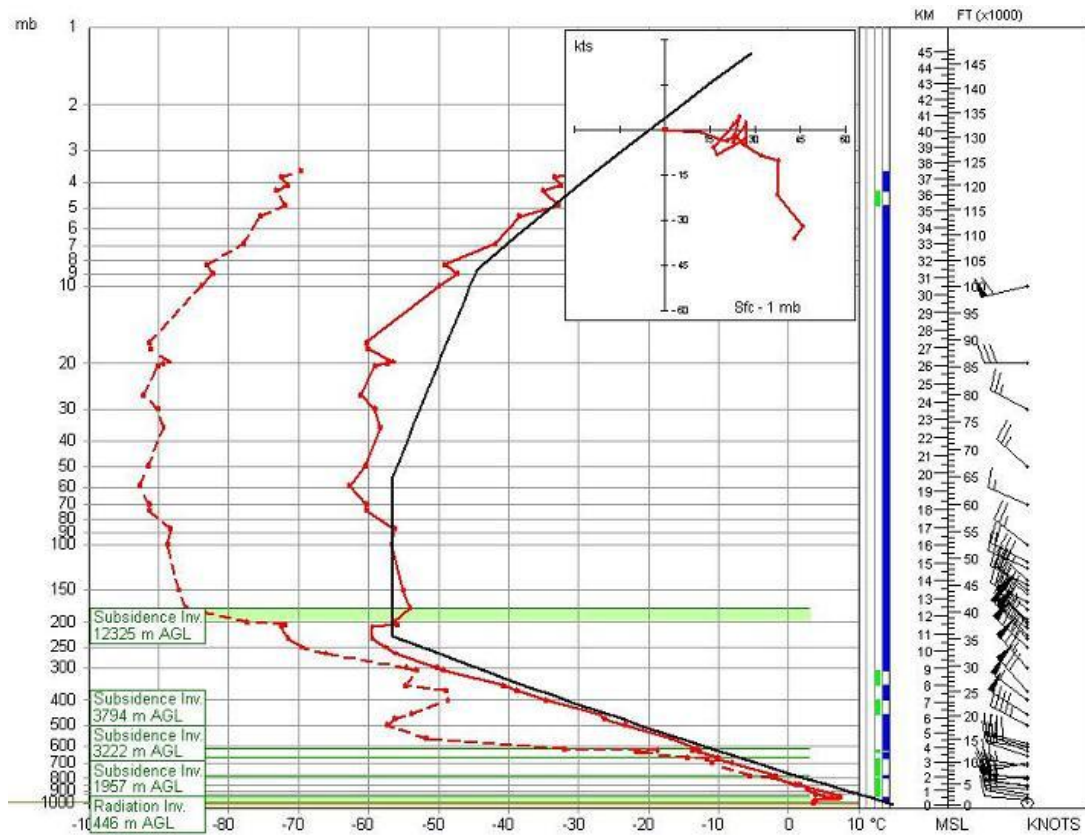
$$A = -R \ln p.$$

This new diagram of (B, A) is called *Emagram*:

x -axis: T (temperature)

y -axis: $-R \ln p$ (natural log of pressure).

Example 1: (<http://www.raob.com/assets/images/RAOBEmagramLg.jpg>)



Example of an Emagram diagram

The Emagram is great for sounding plots reaching above 100 millibars. (Data can be plotted up to the 1 mb level.) It is also best for viewing lapse rate changes, especially as they relate to inversions (shown here as five graphical overlays). This example shows Subsidence and Radiation inversions. RAOB can also analyze and display frontal inversions (see next example). This example also has a mini-hodograph drawn in the upper-right corner of the emagram and the ICAO Standard Atmosphere is plotted as a black solid line. (Inversions can only be plotted when the optional Analytic module is used.)

(From http://www.raob.com/ex_diagrams.php)

Example 2:

<http://www.unidata.ucar.edu/software/idv/docs/userguide/examples/Soundings.html>

d. The Stuve diagram:

[reading assignment]

It is also called the pseudoadiabatic diagram.

On this diagram, we choose $A=p^\kappa$ and $B=T$.

From Poisson's equation,

$$\theta = T (1000/p)^\kappa, \quad \kappa = R_d/c_p = 0.286.$$

Thus,

$$T = \text{const } \theta p^{0.286}$$

Therefore, constant θ line (adiabat) is a straight line on this map ($T, p^{0.286}$).

The main properties of this diagram:

- a) Does not have equal area transformation, so area is not proportional to energy;
- b) Isotherms, isobars, dry adiabats, ws lines are straight lines. Pseudoadiabats are curved.
- c) Adiabats-isotherm angle is approximately 45° .

The Stüve diagram is used by NWS.

e. Refsdal Diagram

[reading assignment]

$$A = -T \ln p$$

$$B = \ln T$$

- (a) Only isotherms are straight lines;
- (b) Adiabats-isotherm angle depends on scale;
- (c) Area is proportional to energy (work).

This thermodynamic diagram is used by U.S. Navy.

g. The Tephigram: [reading assignment]

Again let $B=T$ and $A=R\ln\alpha+f(T)$. We know B-A is an equal-area transformation of α - p . This time, we choose the Poisson's equation to estimate $\ln\alpha$ and set $f(T)$.

Since

$$\frac{T}{\theta} = \left(\frac{p}{1000} \right)^\kappa = \left(\frac{RT}{1000\alpha} \right)^\kappa ,$$

we found that

$$\ln \alpha = (1/\kappa)(\ln \theta - \ln T) + \ln T + \ln R - \ln 1000$$

or

$$R \ln \alpha = c_p \ln \theta + G(T)$$

Choose $F(T) = -G(T)$ then

$$\begin{aligned} A &= c_p \ln \theta , \\ B &= T . \end{aligned}$$

This is called *Tephigram*.

(1) isotherms are straight lines;

- (2) isobars are logarithmic curves (from the Poisson's equation, $\ln\theta = \ln T + \text{const}$). These curves are almost straight within meteorological conditions.
- (3) w_s lines are nearly straight.
- (4) θ_e lines are curved.
- (5) the angle between T and θ is 90° .

h. The Skew T-log p diagram

This diagram represents an attempt to modify the emagram so as to make the isotherm-adiabat angle more nearly 90° .

Set

$$A = T + \kappa \ln p$$

$$B = -R \ln p$$

where κ is a constant to be determined for maximum angle between T and θ .

On this diagram, the isotherm ($T = \text{const}$) satisfy:

$$A = \text{const} + \kappa \ln p$$

or

$$A = \text{const} - (\kappa / R)B$$

Thus,

$$B = -(R / \kappa)A + \text{const}$$

This is a linear equation with constant slope. So the isotherms are straight parallel lines whose slope depend only on the choice of k . Thus, by carefully choosing k , one can always make the angle between T and θ nearly 90° .

It can be shown that:

a) Area on *skew T-log p* diagram is proportional to energy.

To show this, one needs to prove

$$\left[\frac{\partial A}{\partial \alpha} \right]_B = \left[\frac{\partial P}{\partial B} \right]_\alpha$$

$$\left[\frac{\partial A}{\partial \alpha} \right]_B = \left[\frac{\partial(T + \kappa \ln p)}{\partial \alpha} \right]_{-R \ln p} = \left[\frac{\partial(p\alpha / R + \kappa \ln p)}{\partial \alpha} \right]_{-R \ln p} = \frac{p}{R}$$

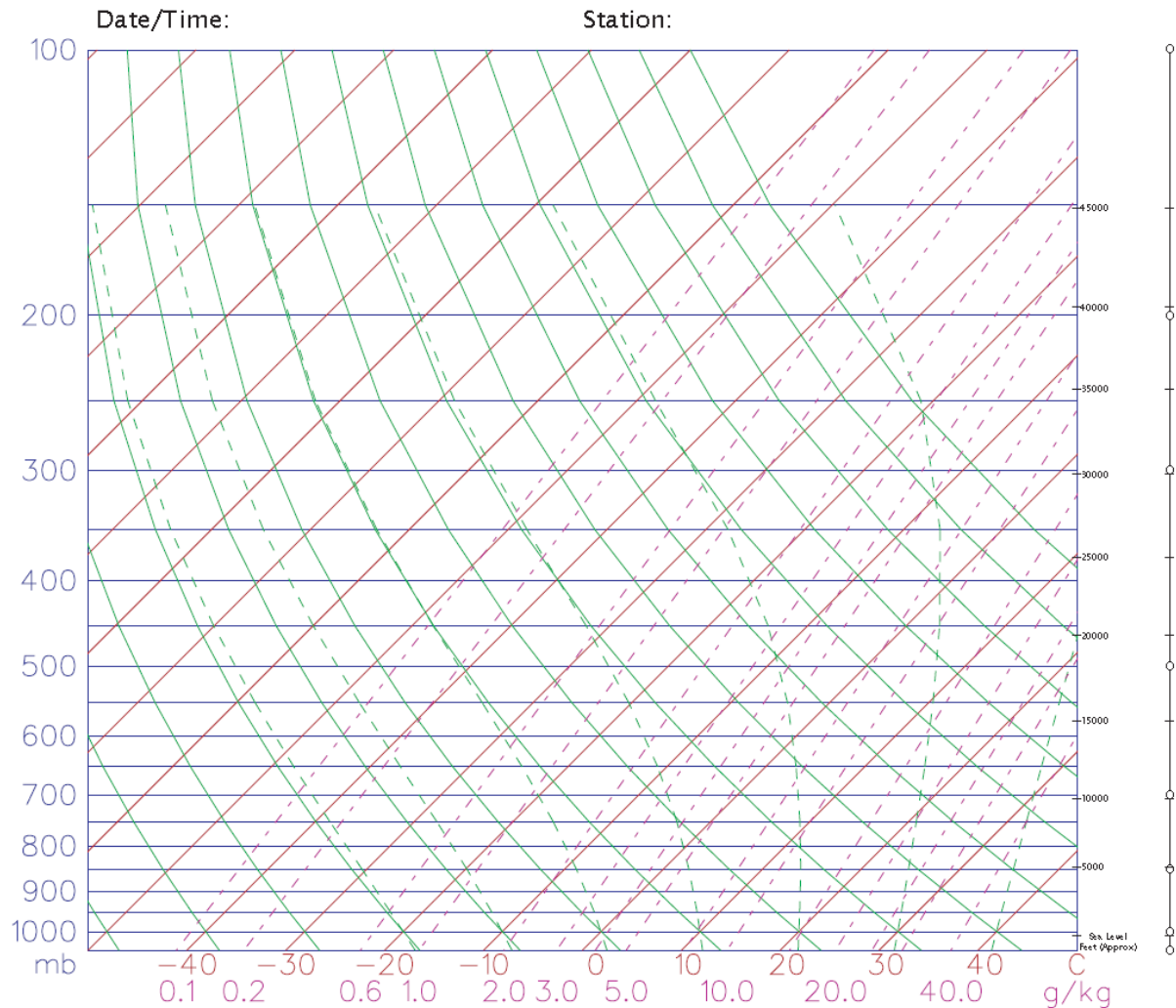
$$\left[\frac{\partial p}{\partial B} \right]_\alpha = \left[\frac{\partial p}{\partial(-R \ln p)} \right]_\alpha = -\frac{1}{R} \left[\frac{\partial p}{\partial(\ln p)} \right]_\alpha = -\frac{p}{R}$$

The only difference between $(\partial A / \partial \alpha)|_B$ and $(\partial p / \partial B)|_\alpha$ is the sign which only affects the direction of the thermodynamic process.

b) Isotherms, isobars, and w_s (*saturation mixing ratio*) lines are straight lines. Dry adiabats are slightly curved lines. Pseudoadiabats are markedly curved lines.

c) Adiabats-isotherm angle is close to 90° .

The *skew T- log p* diagram is used by the *Air Force* and is the most popular one used in the meteorology community.



http://meteora.ucsd.edu/wx_pages/stuff/Blank_Skew-T.pdf

Resources:

- (1) Blank skew T-log p diagram
http://meteora.ucsd.edu/wx_pages/stuff/Blank_Skew-T.pdf
- (2) Understanding the Sounding from Skew T-log p diagram:
http://www.atmos.millersville.edu/~lead/SkewT_Home.html.

- (3) RAP real time weather data <http://www.rap.ucar.edu/weather/upper/>
- (4) Upper-air sounding plots on skew T-log p:
http://weather.unisys.com/upper_air/skew/index.php

i. In-Class Lab: Plot Skew T-log p diagram

Interactive *Skew T-log p* diagram:

<http://www.meted.ucar.edu/mesoprim/skewt/>

Other resources

Thermodynamic Diagrams of Current Soundings

- **Skew-T Diagrams:**
 - from OhioState: [explanation](#) , [soundings & indices for all US & Canadian sites](#) .
 - from NCAR: [soundings, indices & hodographs for all US & Canadian sites](#) .
 - from UNISYS: [explanation](#) , [soundings, indices & text for US sites](#) .
 - from FSU: [soundings & indices for N. America](#) .
 - from TAMU: [soundings for N. America](#) .
 - from NIU: [soundings & indices for N. America](#) .
 - from UWy: [soundings & indices for N. America](#) .
 - from EC via UNBC: [vertical profiles](#) for selected stations in W. Canada
- **Emagram / Pseudoadiabatic Chart:**
 - from U.Wash: [UIL, OTX, SLE, MFR](#) .
 - from NIU: [soundings in N. America](#) .
- **Stuve Diagrams**
 - from AMS DataStreme Project: [blank diagram](#) , [soundings for selected US sites](#) .
 - from FSU: [soundings](#) .
 - from NIU: [soundings in N. America](#) .
 - from UWy: [soundings & indices for N. America](#) .

- **Tephigrams**
 - selected sites in Canada: [Edmonton AB](#) , [Alert NWT](#) , [Montreal PQ](#) .
- ACARS/AMDARS sounding data from commercial aircraft: [NOAA FSL](#) , .
- **Hodographs** of wind:
 - from [FSU](#) .
- Plots soundings on the fly (world-wide) :
 - UWy: [soundings](#) (**Skew-T, Stuve, hodograph**) by clicking on map.
 - with [explanations of indices](#) .
 - NIU : [soundings](#) (**Skew-T, emagram, Stuves, or hodographs**) by entering stn ID.
- Arctic soundings ([U Wisc.-Milwaukee](#))
- Archived sounding data ([FSL](#))
- **Station IDs** for Rawinsonde sites:
 - Map showing Station IDs in any continent ([UWy](#))
 - Map showing Station IDs (letter & number IDs) N. America ([FSU](#))
 - Map showing NWS sounding sites ([NWS](#))
 - List of Station ID names and numbers in N. America ([FSU](#))
 - Station Look-up from [NWS](#) .

Blank Thermo Diagrams (useful for hand plotting)

- Blank graphs, useful for hand-plotting of real soundings:
 - **skew-T** [gif1](#) / [gif_color](#) / [pdf_color](#) (from Riddle, UCSD/Scripps)
 - **tephigram** [gif](#) / [pdf](#) (from Evans, U. Colorado)
 - **Stuve** [gif1](#) / [gif2](#) / [pdf2](#) (from R. Fovell, UCLA)
 - **emagram** [gif1](#) / [pdf1](#) / [gif3](#) / [tiff3](#) / [pdf3](#)(from R. Stull, UBC)
 - emagram (zoomed for PBL) [gif](#) / [tiff](#) / [pdf](#) (from R. Stull, UBC)
 - **theta-Z** [gif](#) / [pdf](#) (from R. Stull, UBC)
 - theta-Z (zoomed for PBL) [gif](#) / [tiff](#) / [pdf](#) (from R. Stull, UBC)
- Sounding data in ascii **text** form (for plotting on blank thermo diagrams)
 - from: [AMS](#) , [UIL](#) , [UWy](#) , [FSU](#) ,

Tutorials and Manuals

- Tutorials:
 - What is a radiosonde? [Brief Intro](#) , [historical photos](#) .
 - From the UK: [Comparison](#) of tephigram, skew-T, Stuve thermo diagrams .
 - Tutorial on how to read tephigrams ([UNBC](#) , [U.Leeds UK](#) , [Project Atmos-Canada.pdf](#))

- From FSU: [How to read Upper Air codes](#) .
- USA Handbook #3 on [Rawinsonde and Pibal Observations](#) .
- NWS Rawinsonde [network info](#) .
- Software to plot soundings (from [UCAR](#))

Stability & Severe Weather Indices

- Excellent explanation ([NWS Louisville](#))
- Stability indices and weather radar ([NWS](#))
- Definitions ([Sirvatka COD](#))
- Explanations and Definitions ([Australian Severe Weather Assoc.](#))
- Definitions and Interpretation ([Ohio State U](#)) , and [contour plots](#) .
- Live excellent [contour plot forecasts of many severe storm indices over most of US](#) (Texas A&M U) with [definitions](#) and [explanations](#) how to interpret.
- Live [contour plots](#) of stability indices, and [definitions](#) - (UNISYS)
- Live contour plots of stability indices for [Canada](#) (EC)
- Forecast contour plots of [total-total index and precipitable water](#) (COLA/IGES) .
- Live [vertical profiles](#) (upper air soundings) with tables of indices & [definition](#) - (UNISYS)
- Live [satellite-derived indices](#) (NWS Western Region) and from [CIMSS](#)