ASME 434 Atmospheric Dynamics II Dr. Yuh-Lang Lin
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Lecture Note 10 Diagnose vertical motion based on *Q*-Vectors

In the ω equation, terms B and C have their own physical processes. In practice, when the ω equation is applied to diagnose the vertical motion, there is often a significant cancellation between these 2 terms. In addition, they are not invariant under a Galilean transformation of the zonal coordinates. Thus, an alternative approach, the ***Q***-vector approach has been developed.

On the midlatitude β-plane the quasi-geostrophic prediction equations may be expressed simply as

$$\frac{D_g u_g}{Dt} - f_0 v_a - \beta y v_g = 0 \tag{6.38}$$

$$\frac{D_g v_g}{Dt} + f_0 u_a + \beta y u_g = 0 \tag{6.39}$$

$$\frac{D_g T}{Dt} - \frac{\sigma p}{R}\omega = \frac{J}{c_p} \tag{6.40}$$

These are coupled by the thermal wind relationship

$$f_0 \frac{\partial u_g}{\partial p} = \frac{R}{p}\frac{\partial T}{\partial y}, \quad f_0 \frac{\partial v_g}{\partial p} = -\frac{R}{p}\frac{\partial T}{\partial x} \tag{6.41a,b}$$

or in vector form:

$$\left(f_0 \mathbf{k} \times \frac{\partial \mathbf{V}_g}{\partial p} \right) = \frac{R}{p}\nabla T \tag{6.42}$$

Equations (6.38) – (6.40) with the usage of (6.41a, b) may be manipulated to obtain

$$\sigma\nabla^2\omega + f_o^2\frac{\partial^2\omega}{\partial p^2} = -2\nabla\cdot Q \qquad (6.45)$$

where Q is defined as

$$\mathbf{Q} = -\frac{R}{p}\left|\frac{\partial T}{\partial y}\right|\left(\mathbf{k}\times\frac{\partial \mathbf{V}_g}{\partial x}\right) \qquad (6.55)$$

Since the left side of the above equation is a three-dimensional Laplacian, a simple form of (6.45) for determining the sign (upward or downward) of the vertical motion can be written as

$$\sigma\nabla^2\omega + f_o^2\frac{\partial^2\omega}{\partial p^2} \propto -\omega \propto w \propto -\nabla\cdot Q \qquad (6.46)$$

Or

$$w \propto -\nabla\cdot Q\,. \qquad (6.47)$$

In other words, in a region with convergence of Q vectors ($\nabla\cdot Q < 0$) will produce an upward motion ($w > 0$), and a region with divergence of Q vectors will produce a downward motion.

The direction and magnitude of the **Q** vector at a given point on a weather map can be estimated by referring the motion to a Cartesian coordinate system in which the x axis is parallel to the local isotherm with cold air on the left. Then (6.51) can be simplified to give

$$\mathbf{Q} = -\frac{R}{p}\left(\frac{\partial T}{\partial y}\right)\left(\frac{\partial v_g}{\partial x}\mathbf{i} - \frac{\partial u_g}{\partial x}\mathbf{j}\right)$$

where we have again used the fact that $\partial u_g/\partial x = -\partial v_g/\partial y$. From the rules for cross multiplication of unit vectors, the above expression for **Q** can be rewritten as

$$\mathbf{Q} = -\frac{R}{p}\left|\frac{\partial T}{\partial y}\right|\left(\mathbf{k}\times\frac{\partial \mathbf{V}_g}{\partial x}\right) \qquad (6.55)$$

Thus, the **Q** vector can be obtained by evaluating the vectorial change of $\mathbf{V}_g$ along the isotherm (with cold air on the left), rotating this change vector by 90° clockwise, and multiplying the resulting vector by $|\partial T/\partial y|$.

In-class lab:

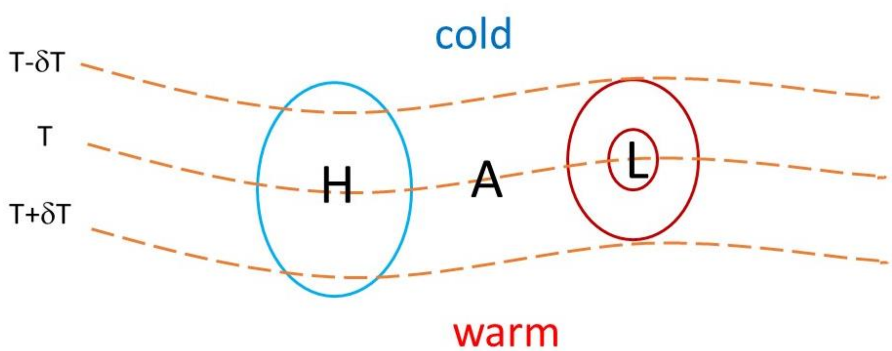
Use the handout lab and based on (6.46) or (6.47) and (6.55), determine the vertical motion at point A by the following the procedure listed below:

(1) Define the x-axis//local isotherm leaving cold air to the left

(2) Determine the Q vectors at the high (H) and low (L) centers by

- (i) Find $\partial V_g / \partial x$ from 2 adjacent points (denoted by "x") near H and L centers along the isotherms. You may assume V_g is along the isobars.
- (ii) Rotating the $\partial V_g / \partial x$ clockwise by 90° to get Q vectors at H and L.

(3) Determine the vertical velocity at A by $w \propto -\nabla \cdot Q$.



Homework: Determine the vertical motion at the entrance region of a jet streak.