ASME 434 Atmospheric Dynamics
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Lecture 8 Quasi-Geostrophic (QG) Prediction of Geopotential Tendency

8.1 Geopotential Tendency Equation

(Equation editor: $D/Dt = \partial/\partial t + u\partial/\partial x$)

Purpose: To derive a prognostic equation for predicting geopotential tendency.

- Based on the hydrostatic equation

$$\frac{\partial \phi}{\partial p} = -\frac{RT}{p} \tag{6.2}$$

We have

$$T = -\frac{p}{R}\frac{\partial \phi}{\partial p}$$

Substituting it into the QG thermodynamic equation

$$\frac{\partial T}{\partial t} = -u_g \frac{\partial T}{\partial x} - v_g \frac{\partial T}{\partial y} + \left(\frac{\sigma p}{R}\right)\omega + \frac{J}{c_p} \tag{6.13}$$

leads to

$$\frac{\partial \chi}{\partial p} = -V_g . \nabla \frac{\partial \phi}{\partial p} - \sigma\omega - \frac{\kappa J}{p} \qquad (6.22)$$

where $\kappa = R/c_p$.

- Equation (6.22) is also called "hydrostatic thermodynamic equation".

Q: What is the physical meaning of individual terms of (6.22)?

- Equations (6.22) and the QG vorticity equation (derived in Ch.7.2 of Holton)

$$\frac{\partial \zeta_g}{\partial t} + u_g \frac{\partial \zeta_g}{\partial x} + v_g \frac{\partial \zeta_g}{\partial y} = f_0 \frac{\partial \omega}{\partial p} - \beta v_g \qquad (6.18)$$

or

$$\frac{1}{f_0}\nabla^2 \chi + u_g \frac{\partial}{\partial x}\left(\frac{1}{f_0}\nabla^2 \phi\right) + v_g \frac{\partial}{\partial y}\left(\frac{1}{f_0}\nabla^2 \phi\right) = f_0 \frac{\partial \omega}{\partial p} - \beta v_g \qquad (6.18)'$$

form a closed set of equations of ϕ and ω since

$$\zeta_g = \frac{1}{f_0}\nabla_p^2 \phi$$

(6.13)

- Eliminate ω ⇒ geopotential tendency (χ) equation
  ⇒ To predict geopotential height tendency
- Eliminate χ ⇒ Omega (ω) equation
  ⇒ To diagnose vertical motion

The geopotential tendency equation can then be derived

$$\left[\nabla^2 + \frac{\partial}{\partial p}\left(\frac{f_o^2}{\sigma}\frac{\partial}{\partial p}\right)\right]\chi = -f_o V_g . \nabla\left(\frac{1}{f_o}\nabla^2\phi + f\right) - \frac{\partial}{\partial p}\left[\frac{-f_o^2}{\sigma} V_g \bullet \nabla\left(-\frac{\partial \phi}{\partial p}\right)\right] - (f_o^2 \kappa)\frac{\partial}{\partial p}\left(\frac{J}{\sigma}\right)$$

Term A Term B Term C Term D

(6.23)

Physical meaning of (6.23) may be understood by the following simple form:

$$-\chi \quad \propto \quad -\mathbf{V_g}\cdot\nabla(\zeta_g + f) \quad + \quad \frac{\partial}{\partial z}(-\mathbf{V_g}\cdot\nabla T)$$

or

$$-\chi \quad \propto \quad -\mathbf{V_g}\cdot\nabla\zeta_g - \beta v_g + \quad \frac{\partial}{\partial z}(-\mathbf{V_g}\cdot\nabla T)$$

Term B: (1) Relative Vorticity Advection ($-\mathbf{V_g}\cdot\nabla\zeta_g$)

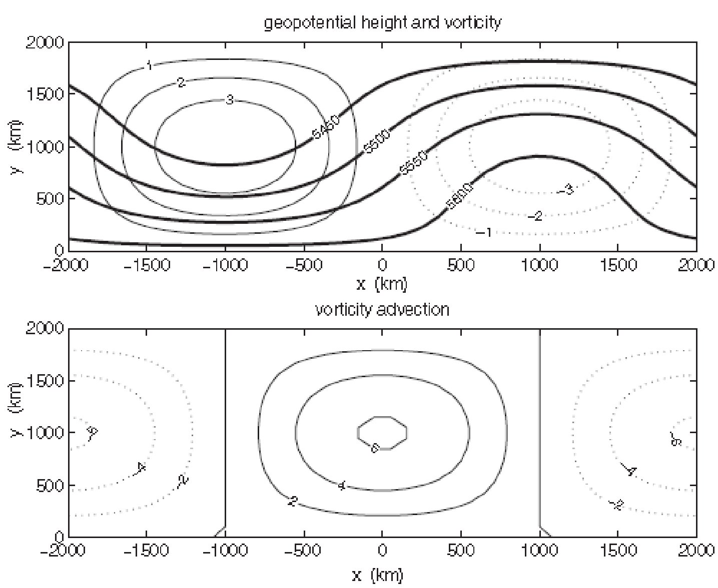
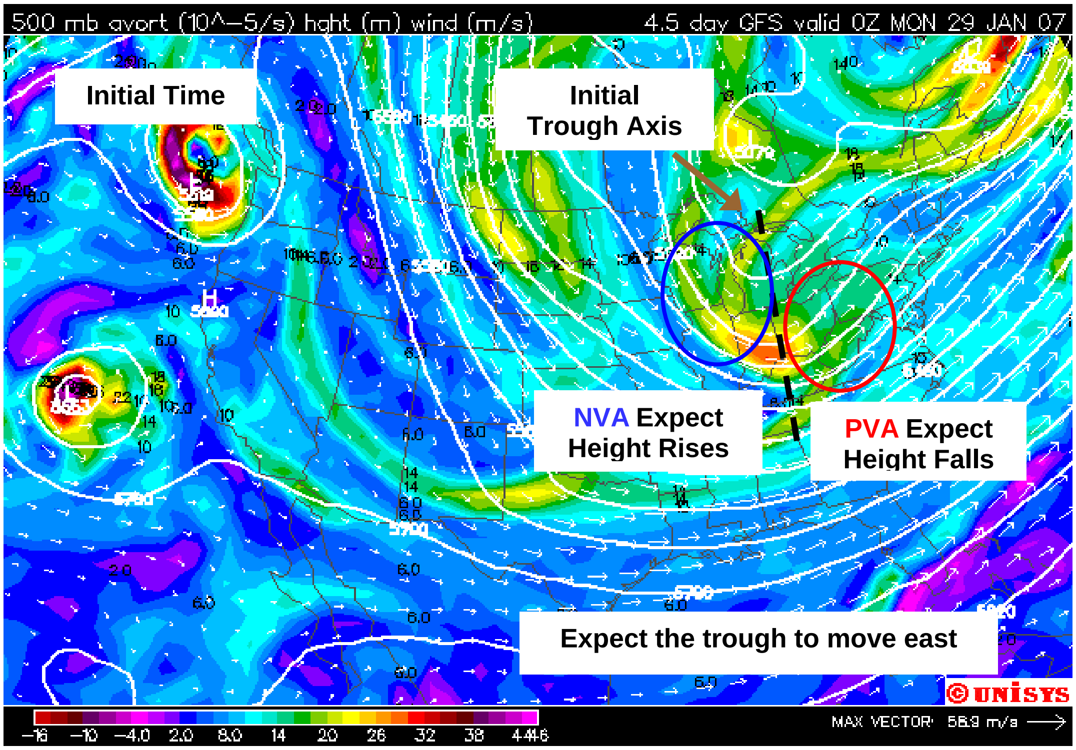


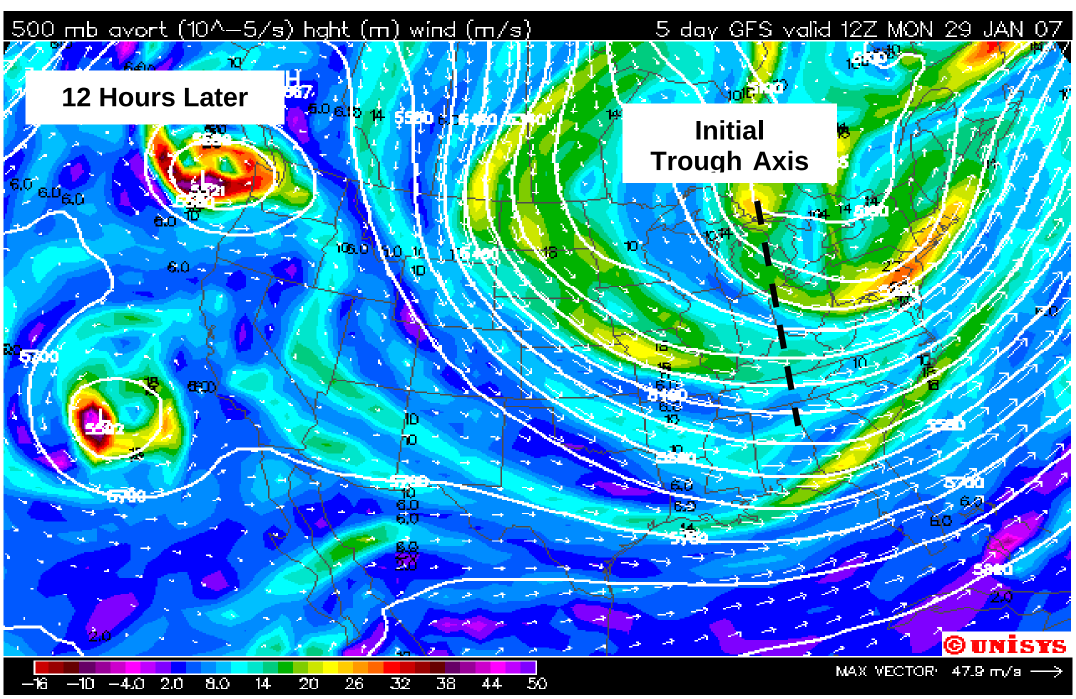
Fig.6.8 (Top) Geopotential height in units of m, and relative vorticity in units of $10^{-5}\,s^{-1}$ for the sinusoidal disturbance of equation (6.20). Here $\Phi_0 = 5.5\times10^4\,m^2\,s^{-2}$, $f_0 = 10^{-4}\,s^{-1}$, $f_0A = 800\,m^2\,s^{-2}$, $U = 10\,m\,s^{-1}$, and $k = l = (\pi/2)\times10^{-6}\,m^{-1}$. (Bottom) Advection of relative vorticity in units of $10^{-10}\,s^{-2}$ for the disturbance shown above.

(2) Planetary Vorticity Advection ($-\beta v_g$): Propagating

westward

Example of Relative Vorticity Advection





Term C: Differential Temperature Advection $\left(\frac{\partial}{\partial z}\left(-V_{\mathbf{g}} \cdot \nabla T\right)\right)$

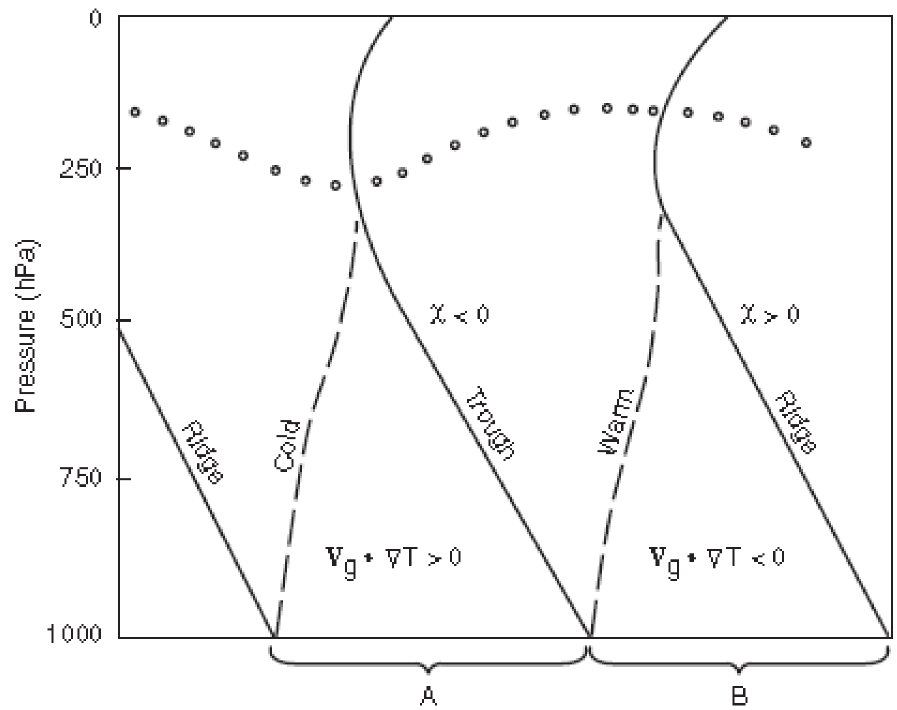
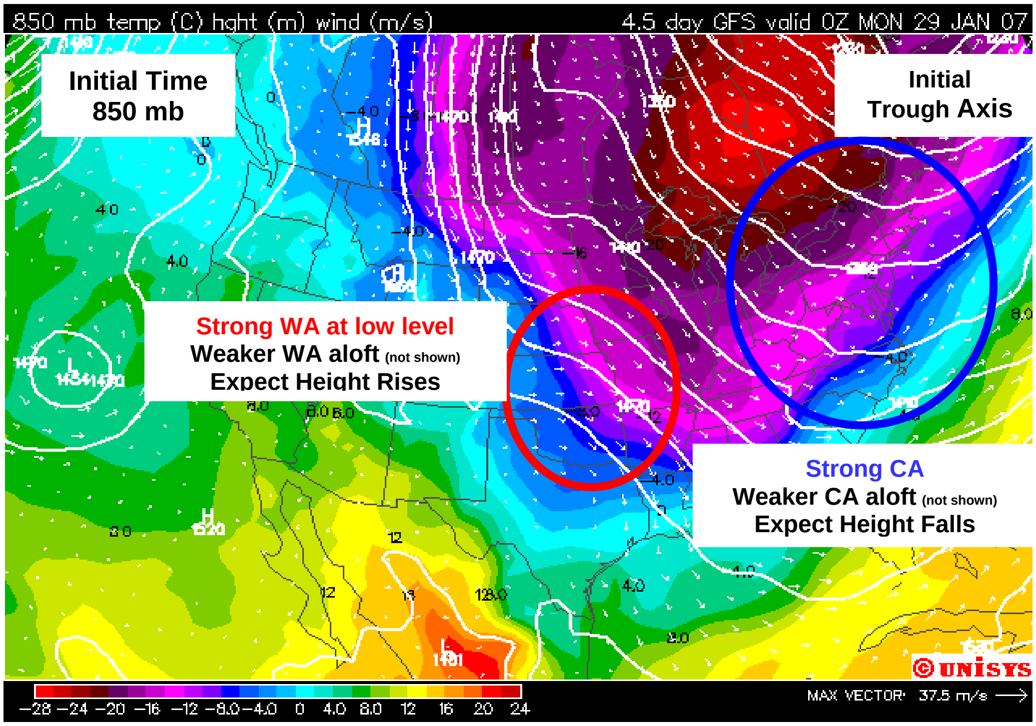
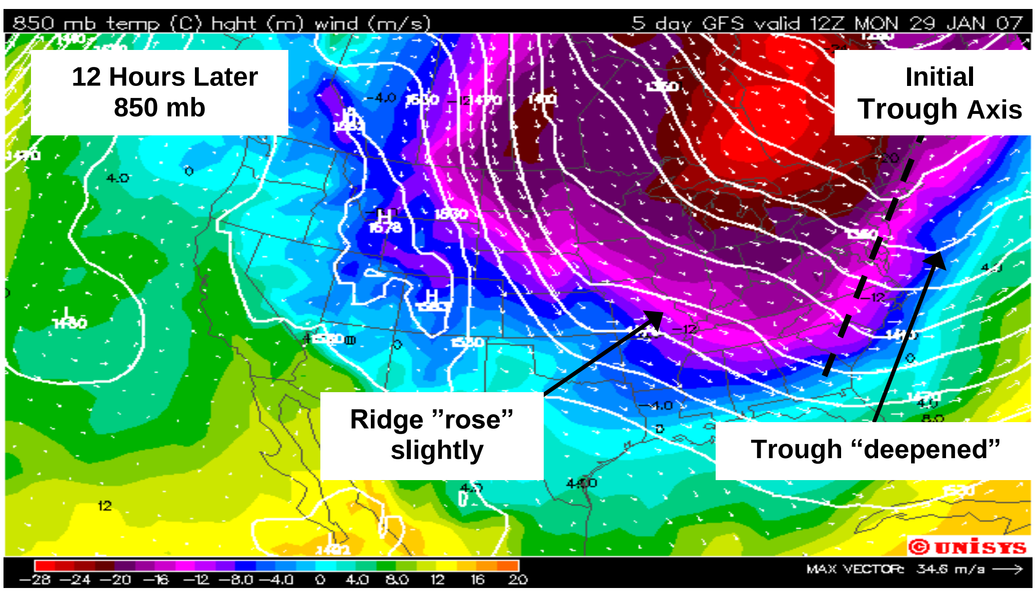


Fig. 6.9 East–west section through a developing synoptic disturbance showing the relationship of temperature advection to the upper level height tendencies. A and B designate, respectively, regions of cold advection and warm advection in the lower troposphere.


850 mb temp (C) hght (m) wind (m/s)
4.5 day GFS valid 0Z MON 29 JAN 07
Initial Time
850 mb
Initial
Trough Axis
Strong WA at low level
Weaker WA aloft (not shown)
Expect Height Rises
Strong CA
Weaker CA aloft (not shown)
Expect Height Falls
© UNISYS
MAX VECTOR: 37.5 m/s


850 mb temp (C) hght (m) wind (m/s)
5 day GFS valid 12Z MON 29 JAN 07
12 Hours Later
850 mb
Initial
Trough Axis
Ridge "rose" slightly
Trough "deepened"
© UNISYS
MAX VECTOR: 34.6 m/s