Lecture Note 9 QG Diagnosis of Vertical Motion

ASME 434 Atmospheric Dynamcis II
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Estimating vertical motion in the atmosphere:

Our Challenge:

- No observations of vertical motion
- Intimately linked to clouds and precipitation
- Actual vertical motions are often several orders of magnitude smaller than their collocated horizontal air motions (w ~ 0.01 - 10 m/s) (u,v ~ 10 - 100 m/s)

- Synoptic-scale vertical motions must be estimated from widely-spaced observations (i.e. the rawindsonde network) every 12-hours

Methods:

- Kinematic Method — Integrate the Continuity Equation. Very sensitive to small errors in winds measurements

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0,$$

to estimate ω at p,

$$\omega(p) = \omega(p_s) + (p_s - p)\left[\frac{\partial \langle u \rangle}{\partial x} + \frac{\partial \langle v \rangle}{\partial y}\right]_p . \qquad \text{H(3.38)}$$

- Adiabatic Method

From the thermodynamic equation
Very sensitive to temperature tendencies (difficult to observe)
Difficult to incorporate impacts of diabatic heating

$$\omega = \frac{1}{S_p}\left[\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right]. \qquad \text{H(3.41)}$$

- QG Omega Equation

Least sensitive to small observational errors
Widely believed to be the best method

The QG Omega Equation:

- We can also derive a ***single*** diagnostic equation for ω by, again, combining our vorticity and hydrostatic thermodyamic equations (the height-tendency versions from before):

$$\frac{1}{f_0}\nabla^2\chi + u_g\frac{\partial}{\partial x}\left(\frac{1}{f_0}\nabla^2\phi\right) + v_g\frac{\partial}{\partial y}\left(\frac{1}{f_0}\nabla^2\phi\right) = f_0\frac{\partial \omega}{\partial p} - \beta v_g \qquad (6.18)$$

$$\frac{\partial \chi}{\partial p} = -V_g.\nabla\frac{\partial \phi}{\partial p} - \sigma\omega - \frac{\kappa J}{p} \qquad (6.22)$$

- To do this, we need to eliminate the height tendency (χ) from both equations

Step 1: Apply the operator $f_0\frac{\partial}{\partial p}$ to the vorticity equation (6.18)

Step 2: Apply the operator ∇^2 to the thermodynamic equation (6.22)

Step 3: Subtract the result of Step 1 from the result of Step 2

After some math, we get the resulting diagnostic equation.

The QG Omega Equation:

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{-\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[-\mathbf{V_g}\cdot\nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} \underbrace{- \frac{R}{\sigma p}\nabla_p^2\left(-\mathbf{V_g}\cdot\nabla_p T\right)}_{\text{Term C}}$$

Term A **Term B** **Term C**

- To obtain an ***actual value*** for ω (the ideal goal), we would need to compute the forcing terms (Terms B and C) from the three-dimensional wind and temperature fields, and then invert the operator in Term A using appropriate boundary conditions
- Again, this is not a simple task (*forecasters don't do this*).

- Rather, we can ***infer the sign and relative magnitude*** of ω through simple inspection of the three-dimensional absolute vorticity and temperature fields (*forecasters do this all the time*)
- Thus, let's examine the physical interpretation of each term.

The QG Omega Equation:

$$\underbrace{\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{-\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[-\mathbf{V_g}\cdot\nabla\left(\zeta_g + f\right)\right]}_{\text{Term B}} \underbrace{- \frac{R}{\sigma p}\nabla^2\left(-\mathbf{V_g}\cdot\nabla_p T\right)}_{\text{Term C}}$$

For sinusoidal disturbances, the above eq. may be roughly simplified to

$$\underbrace{w}_{\text{Term A}} \propto \underbrace{\frac{\partial}{\partial z}\left[-\mathbf{V_g}\cdot\nabla\left(\zeta_g + f\right)\right]}_{\text{Term B}} \underbrace{- \mathbf{V_g}\cdot\nabla T}_{\text{Term C}}$$

Term A: Local Vertical Motion

- Again, if we incorporate the negative sign into our physical interpretation, which we will do, we can just think of this term as the vertical motion

- Thus, this term is our goal – a qualitative estimate of the deep –layer synoptic-scale vertical motion at a particular location

A Simple Form of the QG Equation:

$$w = \frac{\partial}{\partial z}\left[-V_{\mathbf{g}} \cdot \nabla_p (\zeta_g + f)\right] \; -V_{\mathbf{g}} \cdot \nabla_p T$$

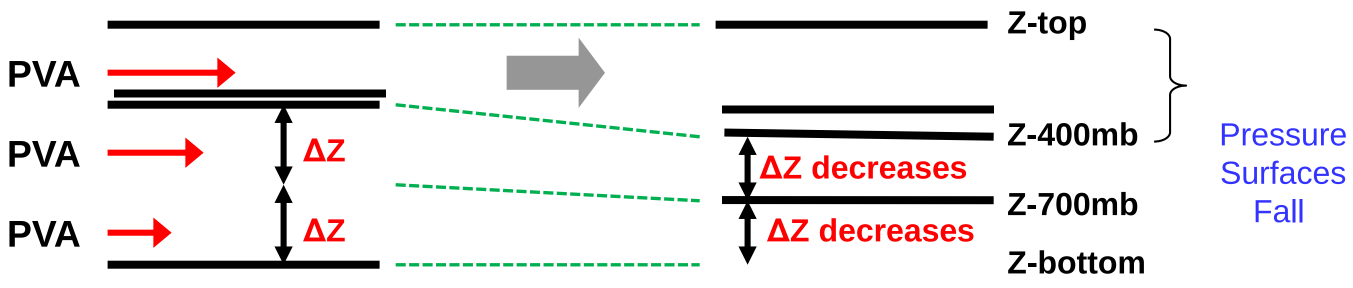
Term A　　　Term B　　　Term C

Term B: Differential Absolute Vorticity Advection

- Recall, positive (relative) vorticity advection (PVA) leads to local height falls

$$\zeta_g = \frac{1}{f_0} \nabla_p^2 \phi \quad \Rightarrow \quad \phi \propto -\zeta_g$$

- Consider a three-layer atmosphere where cyclonic vorticity advection increases with height, or PVA is strongest in the upper layer:



- Hydrostatic balance (and the hypsometric equation) requires ALL changes in thickness (ΔZ) to be accompanied by temperature changes…

A Simple Form of QG Omega Equation:

$$w \propto \frac{\partial}{\partial z}\left[-V_{\mathbf{g}} \cdot \nabla_p \left(\zeta_g + f\right)\right] - V_{\mathbf{g}} \cdot \nabla_p T$$

Term A Term B Term C

Term B: Change in Absolute Vorticity Advection with "Height"

- These thickness decreases (height falls) were ***not*** a result of temperature changes
- Thus, in order to maintain hydrostatic balance, the thickness decreases must be accompanied by a temperature decrease

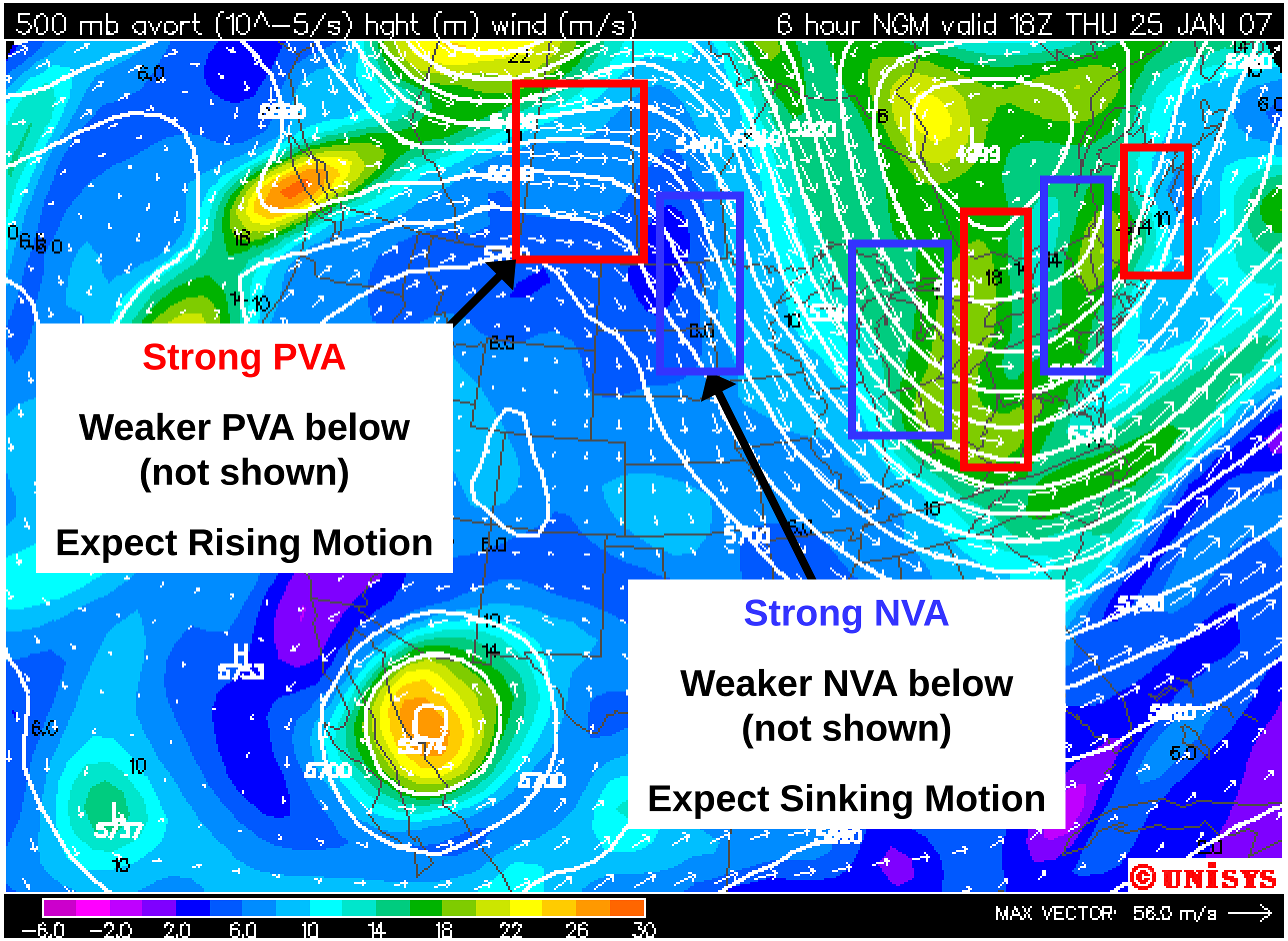
- In the absence of temperature advection and diabatic cooling, only adiabatic cooling associated with rising motion can create this required temperature decrease

- Therefore, an **increase in PVA with height** will induce **rising motion**

QG Diagnosis: Vertical Motion

The BASIC Quasigeostrophic Omega Equation:

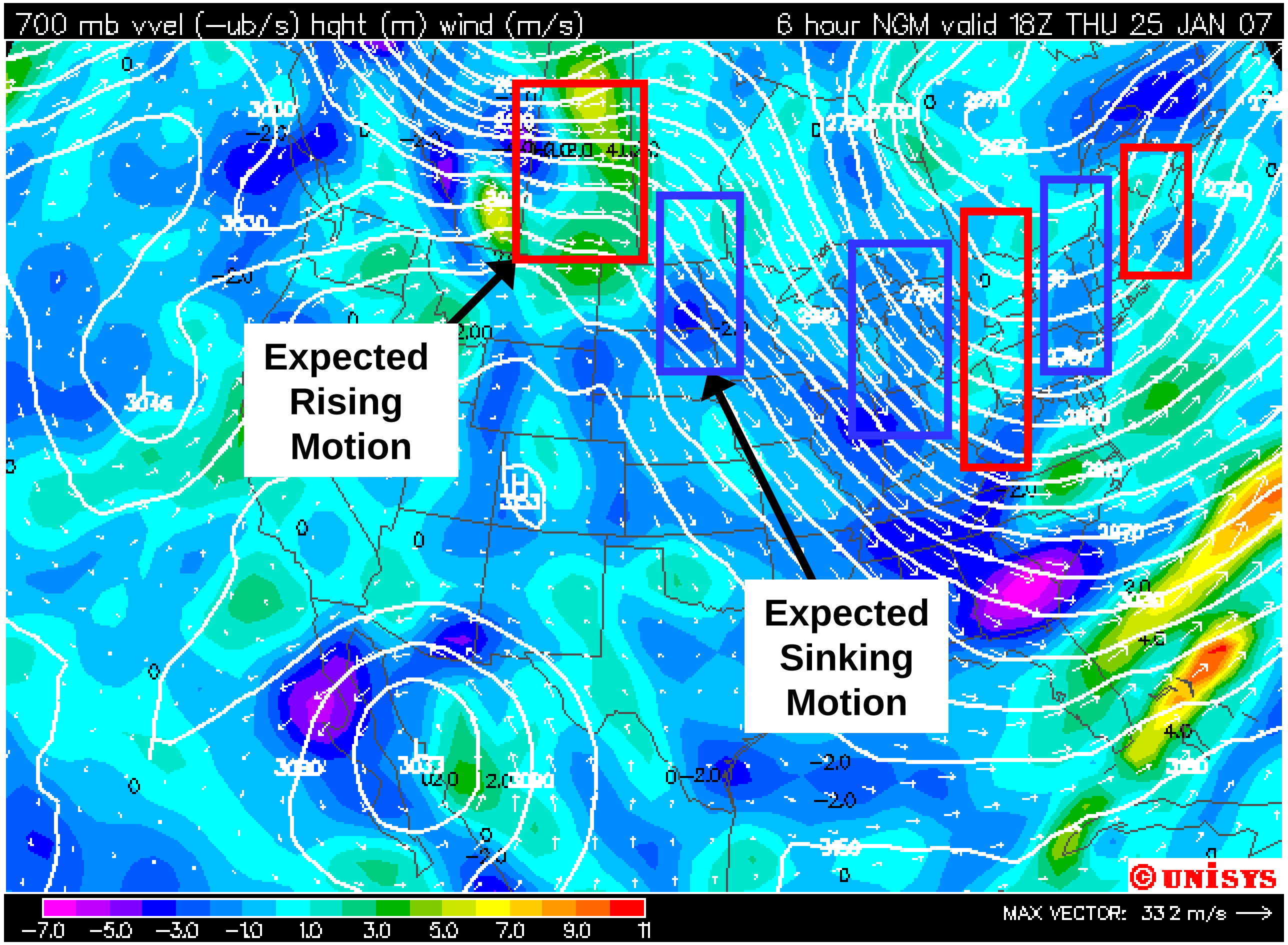
Term B: Change in Relative Vorticity Advection with "Height"



QG Diagnosis: Vertical Motion

The BASIC Quasigeostrophic Omega Equation:

Term B: Change in Absolute Vorticity Advection with "Height"



QG Diagnosis: Vertical Motion

The BASIC Quasigeostrophic Omega Equation:

$$w \quad \propto \quad \frac{\partial}{\partial z}\left[-V_{\mathbf{g}} \cdot \nabla_p \left(\zeta_g + f\right)\right] - V_{\mathbf{g}} \cdot \nabla_p T$$

Term A **Term B** **Term C**

Term C: Horizontal Temperature Advection

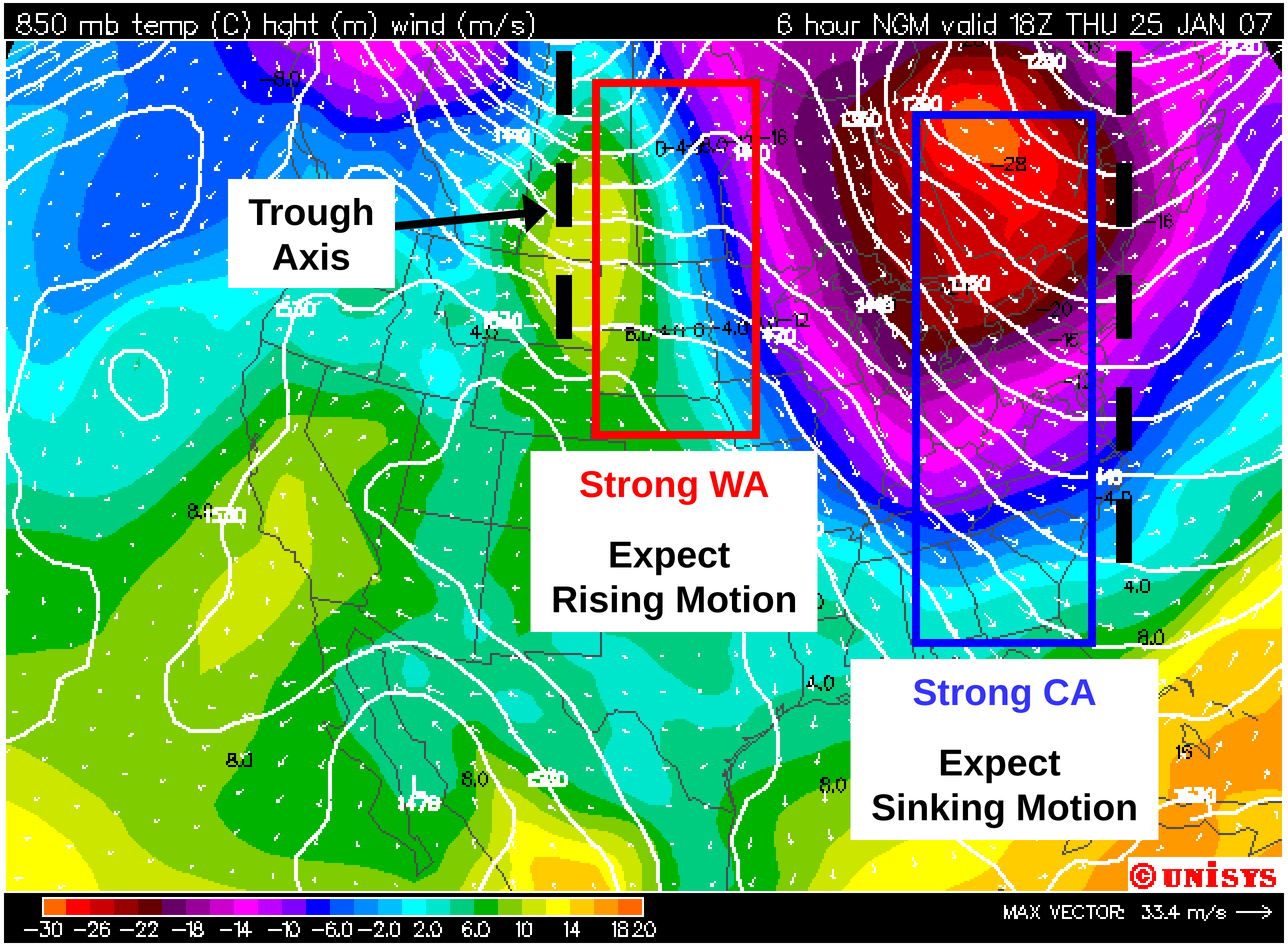
- Warm air advection (**WA**) leads to upward motion

Term C > 0 => Term A > 0

QG Diagnosis: Vertical Motion

The BASIC Quasigeostrophic Omega Equation:

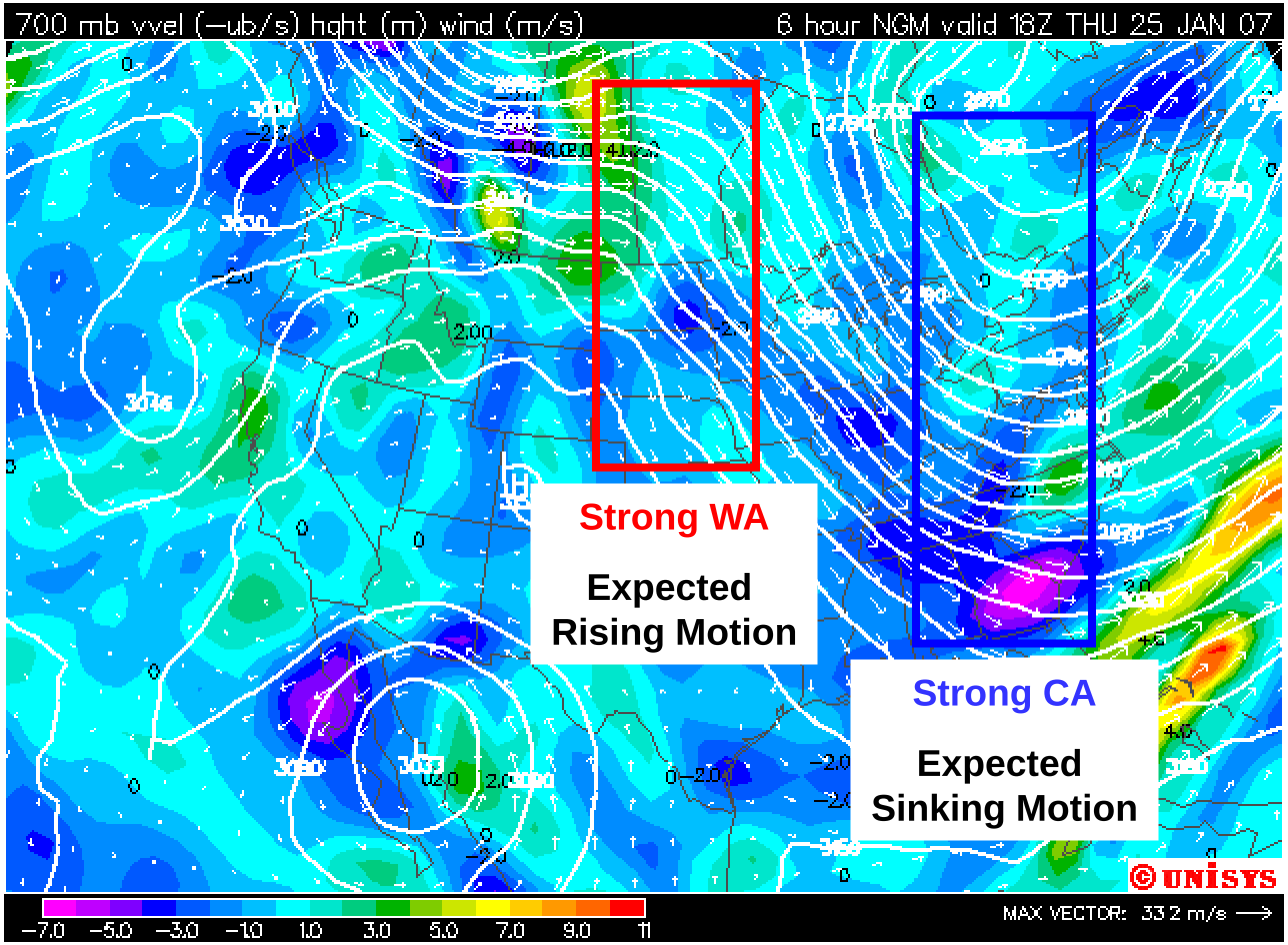
Term C: Horizontal Temperature Advection



QG Diagnosis: Vertical Motion

The BASIC Quasigeostrophic Omega Equation:

Term C: Horizontal Temperature Advection



QG Diagnosis: Vertical Motion

The BASIC Quasigeostrophic Omega Equation:

$$\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = -\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[-\mathbf{V_g}\cdot\nabla_p\left(\zeta_g + f\right)\right] - \frac{R}{\sigma p}\nabla_p^2\left(-\mathbf{V_g}\cdot\nabla_p T\right)$$

Term A **Term B** **Term C**

$$w \propto \frac{\partial}{\partial z}\left[-\mathbf{V_g}\cdot\nabla_p\left(\zeta_g + f\right)\right] - \mathbf{V_g}\cdot\nabla_p T$$

Summary and Application Tips:

- You must consider the effects of both **Term B** and **Term C** at multiple levels
- If large (small) changes in the vorticity advection with height are observed, then you should expect large (small) vertical motions
- The stronger the temperature advection, the stronger the vertical motion
- If WA (CA) is observed at several consecutive pressure levels, expect a deep layer of rising (sinking) motion
- Opposing expectations in vertical motion from the two terms at a given location will alter the total vertical motion pattern

QG Diagnosis: Vertical Motion

Summary and Final Comments:

- The QG omega equation is a **diagnostic** equation:
  - The equation does ***not*** ***predict*** future vertical motion patterns
  - The forcing functions (Terms B and C) do not ***cause*** the expected responses, with an implied time lag between the forcing and the response
  - The responses are *instantaneous*
  - The responses are a direct result of the atmosphere maintaining hydrostatic and geostrophic balance at the time of the forcing
- Use of the QG omega equation in a **diagnostic** setting (forecasting):
  - Diagnose the **synoptic–scale** vertical motion pattern, and assume rising motion corresponds to clouds and precipitation when ample moisture is available
  - Compare to the observed patterns → Infer mesoscale contributions
- Use of the QG omega equation in a **limited prognostic** setting (forecasting):
  - Diagnose the **synoptic-scale contribution** to the total vertical motion, cloud, and precipitation patterns predicted at a future time by a numerical model
  - Help distinguish between regions of persistent precipitation (synoptic scale) and more sporadic precipitation (mesoscale)

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