

Lecture 20 Thermodynamic Diagrams

(Ch.5 of Hess)

20.1 Thermodynamic Diagrams

a. Introduction

- (1) **Purpose:** A thermodynamic diagram is to provide a graphic display of the lines representing the major processes, such as isobaric, isothermal, dry adiabatic, and pseudoadiabatic processes.
- (2) **Fundamental lines include:**
 - a) isobars
 - b) isotherms
 - c) dry adiabats
 - d) pseudoadiabats
 - e) saturation mixing ratio lines.
- (3) **Data to be represented:** the data to be represented are obtained from soundings and consist of sets of values of temperature, pressure, and humidity.
- (4) **Desired characteristics** when designed a thermodynamic diagram:
 - a) Area in the diagram is proportional to work or energy;
 - b) As many as possible of the fundamental lines are straight;

- c) As large as possible of the angle between the isotherms and the dry adiabats.

b. Equal area transformation:

Consider the following two thermodynamic diagrams:

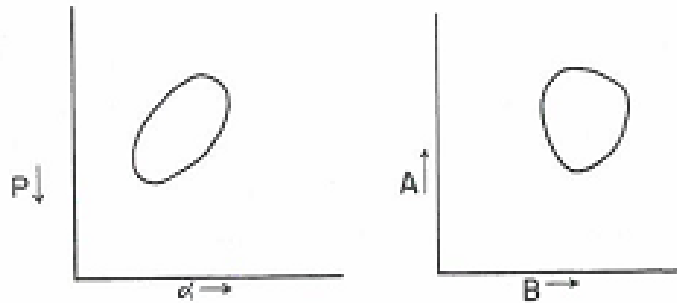


FIG. 5.1. Representation of a cycle on an $\alpha, -p$ diagram and its equal-area transformation to an A, B diagram

In an $\alpha-p$ diagram, the amount of work done for a cyclic process is

$$w = -\oint p d\alpha,$$

which is the area enclosed by the closed curve in the $\alpha-p$ diagram. Since pressure decreases with height, a negative sign is added in front of the closed line integral.

An equal area transformation from $\alpha-p$ coordinate to, say, B-A coordinate requires:

$$w = -\oint p d\alpha = \oint A dB,$$

or

$$0 = \oint p d\alpha + \oint A dB = \oint (p d\alpha + A dB). \quad (21.1.1)$$

[Reading Assignment]

Eq. (21.1.1) implies that $d\phi$ must be an exact differential.

Let

$$d\phi = p d\alpha + A dB,$$

In other words,

$$\phi = \phi(\alpha, B)$$

Thus,

$$d\phi(\alpha, B) = \left[\frac{\partial \phi}{\partial \alpha} \right]_B d\alpha + \left[\frac{\partial \phi}{\partial B} \right]_\alpha dB.$$

Therefore, the sufficient conditions for an equal area transformation are:

$$p = \left[\frac{\partial \phi}{\partial \alpha} \right]_B; \quad A = \left[\frac{\partial \phi}{\partial B} \right]_\alpha.$$

This implies,

$$\frac{\partial p}{\partial B} = \frac{\partial^2 \phi}{\partial \alpha \partial B}; \quad \frac{\partial A}{\partial \alpha} = \frac{\partial^2 \phi}{\partial \alpha \partial B}.$$

Therefore, if

$$\left[\frac{\partial A}{\partial \alpha} \right]_B = \left[\frac{\partial p}{\partial B} \right]_\alpha, \tag{21.1.2}$$

then the area will be equal on the two diagrams.

c. The Emagram:

Consider the case where $B = T$. Then, using (21.1.2), we have

$$\left[\frac{\partial A}{\partial \alpha} \right]_T = \left[\frac{\partial p}{\partial T} \right]_\alpha.$$

Using the equation of state

$$p\alpha = RT,$$

we get

$$\left[\frac{\partial p}{\partial T} \right]_\alpha = \frac{R}{\alpha}.$$

Therefore,

$$\left[\frac{\partial A}{\partial \alpha} \right]_T = \left[\frac{\partial p}{\partial T} \right]_\alpha = \frac{R}{\alpha},$$

or

$$\left[\frac{\partial A}{\partial \alpha} \right]_T d\alpha = \frac{R}{\alpha} d\alpha$$

Integrate this equation, we get

$$A = R \ln \alpha + f(T)$$

where $f(T)$ can be chosen as any function of T .

Take the log of the equation of state, we find

$$\ln \alpha = -\ln p + \ln R + \ln T$$

So,

$$A = -R \ln p + [R \ln R + R \ln T + f(T)]$$

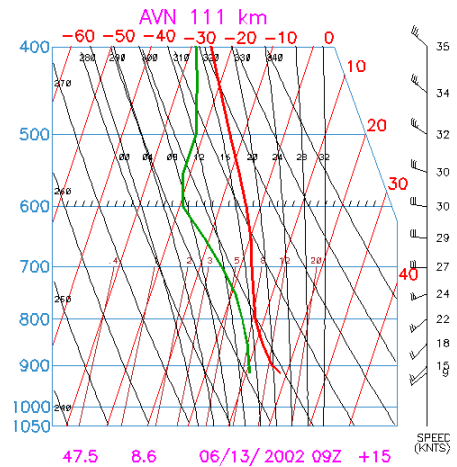
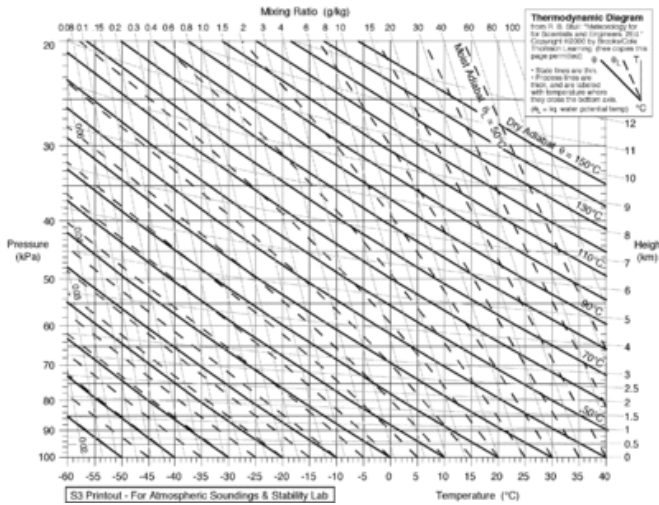
Therefore, if we choose $f(T) = -R \ln R - R \ln T$, then

$$A = -R \ln p$$

This new diagram of (B, A) is called Emagram.

T (temperature) as the x -axis;

$-R \ln p$ (natural log of pressure) as the y -axis;



d. The Skew T-log p diagram

This diagram represents one way to modify the emagram to make the isotherm-adiabat angle large to be nearly 90° .

[Reading Assignment]

Set

$$A = T + \kappa \ln p$$
$$B = -R \ln p$$

where κ is a constant to be determined for maximum angle between T and θ .

On this diagram, the isotherm ($T=\text{const}$) satisfy:

$$A = \text{const} + \kappa \ln p \text{ or } A = \text{const} - (\kappa/R) B$$

Thus,

$$B = - (R/\kappa) A + \text{const}$$

This is a linear equation with constant slope. So, the isotherms are straight parallel lines whose slope depends only on the choice of k . Thus, by carefully choosing k , one can always make the angle between T and θ nearly 90° .

It can be shown that:

1) Area on skew T -Log p diagram is proportional to energy.

To show this, one needs to prove

$$\left(\frac{\partial A}{\partial \alpha}\right)_B = \left(\frac{\partial P}{\partial B}\right)_\alpha$$

$$\begin{aligned} (\partial A/\partial \alpha)|_B &= [\partial(T + k \ln p)/\partial \alpha]|_B - R \ln p \\ &= [\partial(p\alpha/R + k \ln p)/\partial \alpha]|_B - R \ln p = p/R \end{aligned}$$

$$(\partial p/\partial B)|_\alpha = \partial p/\partial(-R \ln p)|_\alpha = -(1/R) \partial p/\partial(\ln p)|_\alpha = -p/R$$

The only difference between $(\partial A/\partial \alpha)|_B$ and $(\partial p/\partial B)|_\alpha$ is the sign which only affects the direction of the thermodynamic process.

b) Isotherms, isobars, and w_s lines are straight lines. Dry adiabats are slightly curved lines. Pseudoadiabats are markedly curved lines.

c) Adiabats-isotherm angle is close to 90° .

The skew T- log p diagram is used by the U.S. Air Force.

A Guide to Skew-T, log-p diagram:

➤ How to read a skew-T, log-p diagram

References:

1. MetEd:

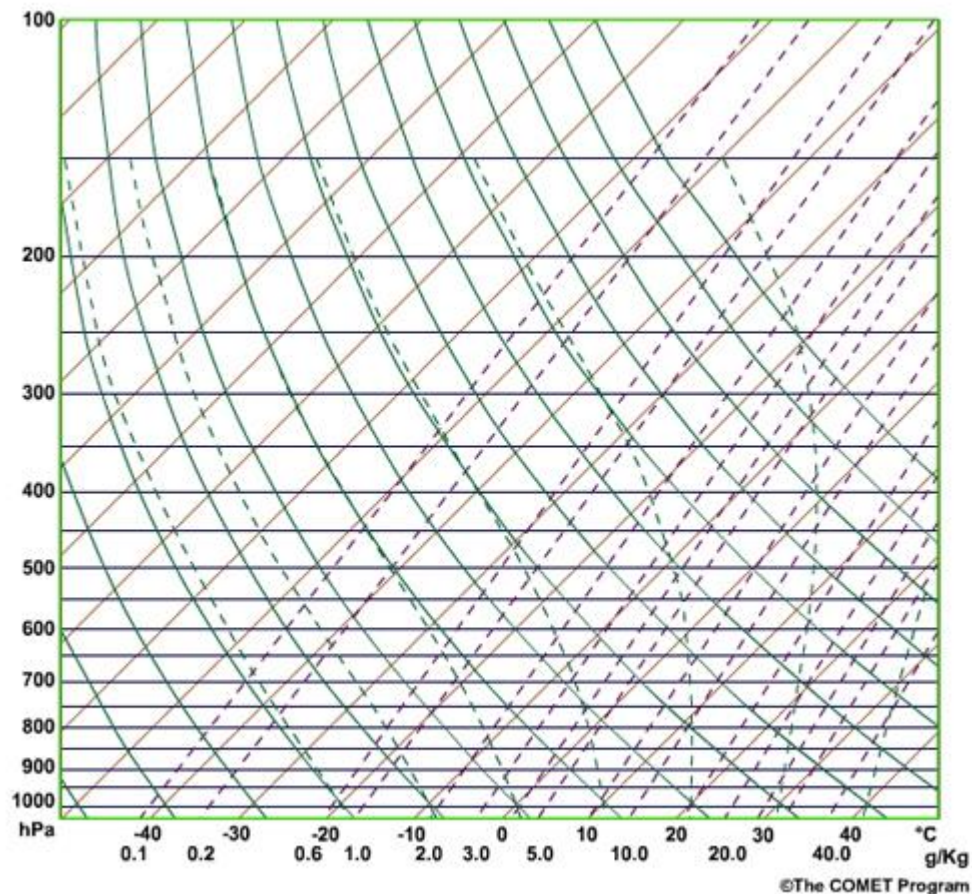
<https://www.meted.ucar.edu/mesoprim/skewt/navmenu.php?tab=1&page=2-6-0&type=flash>

2. PennState METEO 300: <https://www.education.psu.edu/meteo300/node/638>

3. Additional references: [Skew-T, log-p diagram analysis procedure](#); [interactive Skew-T diagram](#)

Let us follow the [MetEd's introduction of skew-T log-p diagram](#) (Skew- T Mastery)

When you first look at a Skew-T, what do you see? There's nothing but a bunch of lines, right? Here is a quick explanation of what all those lines mean.



- **Isobars (D)** -- Lines of equal pressure. Solid brown lines that run horizontally from left to right and are labeled on the left side of the diagram. Pressure is given in increments of 100 mb and ranges from 1050 mb to 100 mb. Spacing between the isobars increases in the vertical because of the log scale that is used to represent pressure.
- **Isotherms** -- Lines of equal temperature. Straight, solid brown lines that slope from the bottom left to the upper right (thus the name skew) across the diagram. Increments are per degree and are labeled for every 5 degrees in units of Celsius. They are labeled at the bottom of the diagram with corresponding temperatures in Fahrenheit.
- **Dry Adiabats (E)** -- Slightly curved, solid brown lines that slant from lower right to upper left. They are labeled every 10 degrees Celsius and indicate the rate of change of temperature in an air parcel of dry air rising or descending adiabatically.
- **Saturation Adiabats (F)** -- Slightly curved, solid green lines sloping from lower right to upper left. They are labeled every 2 degrees Celsius and indicate the rate of change of temperature in a saturated air parcel as it rises pseudo-adiabatically. They become parallel to the dry Adiabats at the top of the chart because of the very low moisture content at those levels and stop at 200 mb.
- **Saturation Mixing Ratio Lines (C)** -- These are the dashed green, slightly curved lines that slope from the lower left to the upper right that represent lines of equal mixing ratio. They are labeled on the bottom of the diagram in grams per kilogram of water vapor. They extend only to 200 mb and the spacing between them decreases as their values increase.
- **Temperature Curve (B)** -- This is the plot of the temperature measurements that were taken from the rawinsonde as it was increasing in height. This curve will always be to the right of the dewpoint curve as you are facing a Skew-T. It is usually drawn in red but can be other colors.
- **Dewpoint Curve (A)** -- This is the plot of the dewpoint measurements increasing with height. This curve will always be to the left of the temperature curve as you are facing a Skew-T. It is usually drawn in green but can be other colors.

➤ **Determining Meteorological Parameters from a Sounding**

The temperature and dewpoint curves that are plotted on the Skew-T are used for much more than simply identifying the temperature and dewpoint at a certain level in the atmosphere.

There are many parameters that are not directly reported on a sounding but that can be calculated by using the lines discussed in Section 1.

Mixing Ratio (w) – the ratio of the mass of [water vapor](#) to the mass of dry air in a sample of moist air. *Units: g of water vapor per kg of dry air (g/kg)*

Procedure: Read the value of the saturation mixing ratio line that crosses the dewpoint curve at that pressure.

Saturation Mixing Ratio (w_s) – the mixing ratio that a sample of air would have if it were saturated. *Units: g of water vapor per kg of dry air (g/kg)*

Procedure: Read the value of the saturation mixing ratio line that crosses the temperature curve at that pressure.

Relative Humidity (RH) – the ratio of the amount of water vapor in a volume of air to the amount of water vapor that the volume could hold if it were saturated *Units: %*

Procedure: Compute from the equation: $RH = (w/w_s) \times 100\% = (e/e_s) \times 100\%$

Vapor Pressure (e) – the part of the total [atmospheric pressure](#) that is contributed by water vapor. *Units: millibars*

Procedure: From the dewpoint curve at a given pressure level, follow the isotherms to the isobar at 622 mb. Read the value of the saturation mixing ratio line that passes through this point to get vapor pressure.

Saturation Vapor Pressure (e_s) -- the [partial pressure](#) that water would add to the total atmospheric pressure if the air was saturated *Units: millibars*

Procedure: From the temperature curve at a given pressure level, follow the isotherms to the isobar at 622 mb. Read the value of the saturation mixing ratio line that passes through this point to get saturation vapor pressure.

Wet Bulb Temperature (T_w) – the lowest temperature to which a volume of air at constant pressure can be cooled by evaporating water into it. *Units: Kelvin*

Procedure: From the dewpoint curve at a given pressure level, draw a line upward along a saturation mixing ratio line. Then from the temperature curve at the given pressure level, draw a line upward along a dry adiabat until it intersects with the first line drawn. From this intersection, follow a saturation adiabat down to the given pressure level. The value of the isotherm running through this point is the wet bulb temperature.

Potential Temperature (θ) – the temperature a parcel of air would have if it were brought dry-adiabatically down to 1000 mb. *Units: Kelvin*

Procedure: From the temperature curve at a given pressure level, follow the dry adiabat that intersects the T curve down to 1000 mb. The isotherm value at this intersection is the potential temperature of the parcel at the given pressure.

Wet Bulb Potential Temperature (θ_w) -- the wet bulb temperature a parcel of air would have if it were brought down to 1000 mb saturation-adiabatically. *Units: Kelvin*

Procedure: Find the wet bulb temperature and follow the saturation adiabat to the 1000 mb isobar. The temperature value at this intersection is the wet- bulb potential temperature.

Equivalent Temperature (T_e) -- the temperature of a parcel if all the moisture was condensed out by a moist adiabatic process using [latent heat](#) to heat the parcel. *Units:* Kelvin
Procedure: From the dewpoint curve at the given pressure level, follow a line upward along a saturation mixing-ratio line. Then from the temperature curve at the given pressure level, follow a line upward along a dry adiabat. The intersection of these two lines is the LCL. From this intersection, follow a saturation adiabat upward to a pressure where the saturation and dry adiabats are parallel; where all the moisture has been condensed out of the parcel. Then follow a dry adiabat down to the original pressure level. The temperature at this level is the equivalent temperature.

Equivalent Potential Temperature (θ_e) – the temperature of a parcel if all the moisture was condensed out by a moist adiabatic process using latent heating to heat the parcel and then the parcel brought dry-adiabatically down to 1000 mb. *Units:* Kelvin
Procedure: Find the equivalent temperature for the given pressure level, then from this point, follow a dry adiabat down to 1000 mb.

Virtual Temperature (T_v) – the temperature at which dry air would have the same [density](#) as moist air at the same pressure. *Units:* Kelvin
Procedure: The virtual temperature can be computed by the equation: $T_v = T + (w/6)$

- Using Skew-T log-p Diagram to determine parameters related to conditional instability

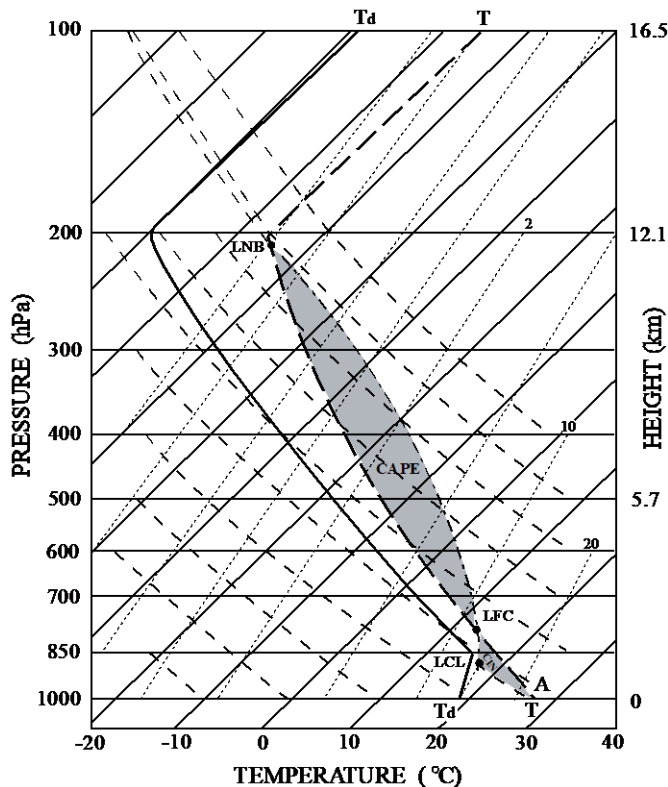


Fig. 7.5: Example of a sounding with conditional instability displayed on a *skew-T log-p* thermodynamic diagram. The lifting condensation level (LCL), level of free convection (LFC), and level of neutral buoyancy (LNB) for the air parcel originated at A are denoted in the figure. The convective available potential energy (CAPE) is the area enclosed by the temperature curve (thick dashed line) and moist adiabat (dot-dashed curve) in between LFC and LNB, while the convective inhibition is the area enclosed the temperature curve and dry adiabat (below LCL) and moist adiabat (above LCL) in between the surface and the LFC. (From Lin 2007, Mesoscale Dynamics)

Lifting Condensation Level (LCL) -- the height at which a parcel of air becomes saturated when it is lifted dry-adiabatically. *Units:* mb

Procedure: From the dewpoint curve at the given pressure level, follow a line upward along a saturation mixing-ratio line. Then from the temperature curve at the given pressure level, follow a line upward along a dry adiabat. The intersection of these two lines is the LCL.

Level of Free Convection (LFC) -- the height at which a parcel of air that is lifted dry or moist adiabatically becomes less dense (warmer) than the surrounding air. *Units:* mb

Procedure: Find the region of positive area on the sounding. The pressure level at the bottom of the positive area that is closest to earth's surface is the LFC. Just below this point, the temperature of the parcel and the temperature of the environment should be equal.

Level of Neutral Buoyancy (Equilibrium Level) (LNB) -- the height where the temperature of a positively buoyant parcel of air becomes equal to that of the surrounding atmosphere and above this level the parcel becomes negatively buoyant.

CAPE (Convective Available Potential Energy) – the maximum energy available on an ascending air parcel. *Units:* Joules per kilogram: (J/kg)

Procedure: To find CAPE on a Skew-T/Log-P Diagram you first must find the level of free convection (LFC). From the LFC, follow the air parcel up along a moist adiabat until it intersects with the temperature curve. This will be the level of neutral buoyancy (LNB). The area to the left of the temperature curve between the LFC and the LNB is CAPE. CAPE is often called Positive Area which is proportional to the amount of [kinetic energy](#) the parcel can gain from the environment.

CIN (Convection Inhibition) – the energy needed to lift an air parcel vertically and pseudoadiabatically from its originating level to its level of free convection (LFC).

Units: J/kg

Procedure: Take the surface dew point temperature and raise it adiabatically. At the same time, take the surface temperature and raise it pseudoadiabatically. Raise these two parcels until they intersect. From the point of intersection, raise the parcel pseudoadiabatically until it crosses the temperature curve. This point is the level of free convection. The area under the LFC and to the right of the temperature curve is CIN. CIN is often called Negative Area which is proportional to the amount of kinetic energy that must be supplied to move the parcel to the LFC.

Convection Condensation Level (CCL) -- the height to which a parcel of air if heated sufficiently from below will rise adiabatically until it is just saturated. This is the height of the base of cumuliform clouds produced by surface heating. *Units:* mb

Procedure: Find the average mixing ratio in the lowest 50 mb and follow this mixing ratio line up to where it intersects the temperature sounding. This point is the height of the CCL.

Convection Temperature (T_c) -- the surface temperature that must be reached to start the formation of [convection clouds](#) by surface heating. *Units:* Kelvin

Procedure: First determine the CCL on the sounding. From the CCL on the temperature curve, follow a dry adiabat downward to the surface. The temperature at this intersection is the convection temperature.

Mixing Condensation Level (MCL) -- the lowest height at which saturation occurs after the complete mixing of the layer. *Units:* mb

Procedure: Follow the mixing ratio line that passes through the surface dewpoint up to where it intersects the temperature sounding. This is the MCL.

e. The Stuve diagram:

[Reading Assignment]

It is also called the pseudoadiabatic diagram.

On this diagram, we choose $A = p^\kappa$ and $B = T$.

From Poisson's equation,

$$\theta = T (1000/p)^\kappa, \quad \kappa = R_d/c_p = 0.286.$$

So,

$$T = C \theta p^{0.286}$$

where C is a constant. Therefore, constant θ line (adiabat) is a straight line on this map $(T, p^{0.286})$.

The main properties of this diagram:

- a) Does not have equal area transformation, so area is not proportional to energy;
- b) Isotherms, isobars, dry adiabats, w_s lines are straight lines. Pseudoadiabats are curved.
- c) Adiabats-isotherm angle is approximately 45° .

The Stuve diagram is used by NWS.

f. Refsdal Diagram:

[reading assignment]

$$A = -T \ln p$$

$$B = \ln T$$

- (a) Only isotherms are straight lines;
- (b) Adiabats-isotherm angle depends on scale;
- (c) Area is proportional to energy (work).

This thermodynamic diagram is used by U.S. Navy.

g. The Tephigram: [reading assignment]

Again let $B=T$ and $A=R\ln\alpha+f(T)$. We know B-A is an equal-area transformation of $\alpha-p$. This time, we choose the Poisson's equation to estimate $\ln\alpha$ and set $f(T)$.

Since

$$\frac{T}{\theta} = \left(\frac{p}{1000}\right)^\kappa = \left(\frac{RT}{1000\alpha}\right)^\kappa,$$

we found that

$$\ln \alpha = (1/\kappa)(\ln \theta - \ln T) + \ln T + \ln R - \ln 1000$$

or

$$R\ln\alpha = c_p \ln\theta + G(T)$$

Choose $F(T) = -G(T)$ then

$$\begin{aligned} A &= c_p \ln\theta \\ B &= T \end{aligned}$$

This is called *Tephigram*, which has the following properties:

- (1) isotherms are straight lines;
- (2) isobars are logarithmic curves (from the Poisson's equation, $\ln\theta = \ln T + \text{const}$).
These curves are almost straight within meteorological conditions.
- (3) w_s lines are nearly straight.
- (4) θ_e lines are curved.
- (5) the angle between T and θ is 90° .