AST 851 Dynamic Meteorology
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Ch. 7.5 Q-Vector and Ageostrophic Circulation

A. Q-Vector

In order to better appreciate the essential role of the divergent ageostrophic motion in quasi-geostrophic flow, it is useful to examine separately the rates of change, following the geostrophic wind, of the vertical shear of the geostrophic wind and of the horizontal temperature gradient.

On the midlatitude β-plane the quasi-geostrophic prediction equations may be expressed simply as

$$\frac{D_g u_g}{Dt} - f_0 v_a - \beta y v_g = 0 \tag{6.38}$$

$$\frac{D_g v_g}{Dt} + f_0 u_a + \beta y u_g = 0 \tag{6.39}$$

$$\frac{D_g T}{Dt} - \frac{\sigma p}{R}\omega = \frac{J}{c_p} \tag{6.40}$$

These are coupled by the thermal wind relationship

$$f_0 \frac{\partial u_g}{\partial p} = \frac{R}{p}\frac{\partial T}{\partial y}, \qquad f_0 \frac{\partial v_g}{\partial p} = -\frac{R}{p}\frac{\partial T}{\partial x} \tag{6.41a,b}$$

or in vector form:

$$\left(f_0 \mathbf{k} \times \frac{\partial \mathbf{V}_g}{\partial p} \right) = \frac{R}{p}\boldsymbol{\nabla} T \tag{6.42}$$

Equations for the evolution of the thermal wind components are obtained by taking partial derivatives with respect to p in (6.38) and (6.39), multiplying through by f_0, and applying the chain rule of differentiation in the advective part of the total derivative to obtain

The direction and magnitude of the **Q** vector at a given point on a weather map can be estimated by referring the motion to a Cartesian coordinate system in which the x axis is parallel to the local isotherm with cold air on the left. Then (6.51) can be simplified to give

$$\mathbf{Q} = -\frac{R}{p}\left(\frac{\partial T}{\partial y}\right)\left(\frac{\partial v_g}{\partial x}\mathbf{i} - \frac{\partial u_g}{\partial x}\mathbf{j}\right)$$

where we have again used the fact that $\partial u_g/\partial x = -\partial v_g/\partial y$. From the rules for cross multiplication of unit vectors, the above expression for **Q** can be rewritten as

$$\mathbf{Q} = -\frac{R}{p}\left|\frac{\partial T}{\partial y}\right|\left(\mathbf{k} \times \frac{\partial \mathbf{V}_g}{\partial x}\right) \tag{6.55}$$

Thus, the **Q** vector can be obtained by evaluating the vectorial change of $\mathbf{V}_g$ along the isotherm (with cold air on the left), rotating this change vector by 90° clockwise, and multiplying the resulting vector by $|\partial T/\partial y|$.

Based on (6.55), the Q vector can be obtained by:

(1) Evaluating the vectorial change of Vg along the isotherm (with cold air to the left,

(2) Rotating the resulting change vector by 90o clockwise, and

(3) Multiplying the resulting vector by (R/p) $|\partial T / \partial y|$.

In the situation shown in Fig. 6.14, the geostrophic flow is confluent so that the geostrophic wind increases eastward along the isotherms. In this case the vectorial

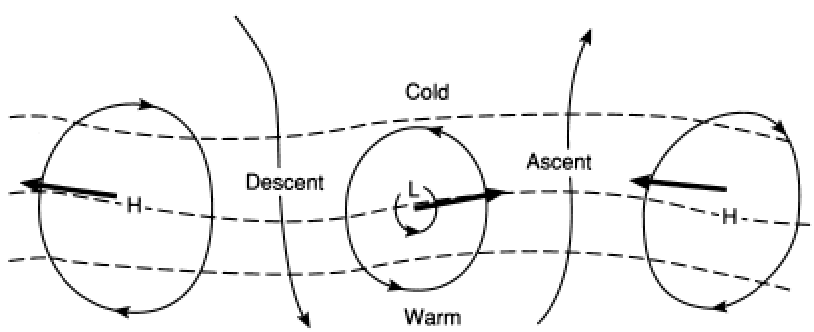
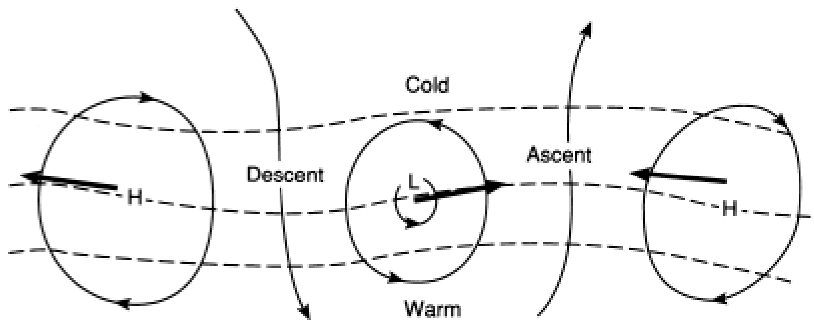


Fig. 6.13 **Q** vectors (bold arrows) for an idealized pattern of isobars (solid) and isotherms (dashed) for a family of cyclones and anticyclones. (After Sanders and Hoskins, 1990.)



Cold
Descent
L
Ascent
H
H
Warm