Chapter 14 Parameterizations of physical processes

14.1 Reynolds averaging

In the previous two chapters, we have described numerical methods for approximating the primitive equations and the setup for numerical models, such as grid systems, and initial and boundary conditions. As demonstrated in Section 13.5, a mesoscale model may be developed to simulate a simple geophysical fluid system, such as a stratified, inviscid flow over topography. However, in order to apply this type of geophysical fluid dynamics model to simulate mesoscale atmospheric phenomena, some important physical processes, such as boundary layer processes, moist processes, and radiative transfer processes, need to be represented or parameterized in the model.

In order to numerically integrate the governing differential equations in a limited area, a numerical method, such as a finite difference method, spectral method or finite element method, must be used to approximately represent the atmospheric motion and processes by the dependent variables at grid points or elements. The approximations limit the explicit representation of atmospheric motions and processes to a scale smaller than that for the grid interval, truncated wavelength, or finite element. For example, large-scale disturbances may cascade down to mesoscale, then further down to the smallest turbulent eddies responsible for viscous dissipation in the atmosphere. If the subgrid-scale disturbances are not appropriately represented by the grid point values, they may cause nonlinear aliasing and nonlinear numerical instability, as discussed in Chapter 13. An obvious way to resolve the problem is to provide sufficient resolution in a numerical model, so that the model can explicitly simulate any significant small-scale motions and processes. For example, *direct numerical simulation* (*DNS*) has been

developed to numerically simulate turbulent motions by fluid dynamicists, in which the time-dependent Navier-Stokes equations with explicit terms for molecular diffusion are integrated numerically to obtain the solution, without making any turbulence parameterizations. The finest scales of the simulation are determined by the balance between nonlinear advection and viscous diffusion, i.e. the *Reynolds number* (*Re = UL/ν,* where *U* and *L* are the characteristic velocity and length scales, respectively, and ν the kinematic viscosity coefficient) of the flow. A typical value of kinematic viscosity for the air in the lower atmosphere is $1.5 \text{x} 10^{-5} m^2 s^{-1}$. When Re>>1, changes in motion by advection are much more important than the dissipation due to molecular viscosity. In this type of turbulent flow, a *turbulent Reynolds number* is more appropriately used to describe the characteristic of the flow, in which the kinematic viscosity coefficient is replaced by the *turbulent exchange coefficient*. Boundary layers encountered in engineering practice have a fairly large Reynolds numbers ranging from 10^3 to 10^6, while the atmospheric boundary layers developing over most natural surfaces are characterized by even larger Reynolds numbers (10^6 to 10^9). The higher Reynolds number flows have also been observed in the free atmosphere, such as within cumulus cloud and in wave breaking regions.

DNS requires the whole range of spatial and temporal scales of the turbulence to be resolved by the grid interval (Δx), from the smallest dissipative scale (L_ε), where L_ε is approximately equal to $(\nu^3/\varepsilon)^{1/4}$ and ε is the kinetic energy dissipation. To satisfy these conditions, the number of grid intervals N in the grid direction must satisfy $N\Delta x > L$ and $\Delta x < L_\varepsilon$, where L is the integral scale. Since $\varepsilon \approx U^3/L$, a three-

dimensional DNS will require a number of grid intervals $N^3 \geq Re^{9/4}$. Thus, the computational cost for DNS is extremely high. With current computing power, it is unrealistic to apply DNS to mesoscale atmospheric modeling. On the other hand, even when the needed computing power is available; we still have to be careful in using the detailed information about small-scale turbulent motions and processes with sizes that cannot be resolved by available observational systems. Since these processes are not well understood at the present time, the governing equations of fluid motion cannot describe them accurately.

The second approach is to numerically integrate the Reynolds-averaged Navier-Stokes (*RANS*) equations of the mean motion. The ensemble properties of all time fluctuations in a turbulent flow are described by a turbulence closure. In this approach, the subgrid-scale motions and processes are parameterized. The parameterization approach gives a less detailed representation than the explicit representation (DNS), but it is more practical in terms of computing cost and may be sufficiently accurate for many mesoscale models since it considers grid interval and initial data, among other factors.

A third approach in numerically simulating turbulent flows is to simulate *large turbulent eddies* explicitly, while the unresolved subgrid scale motions associated with smaller turbulent eddies are either ignored or parameterized. In this type of *large-eddy simulations* (*LES*), the large turbulent eddies explicitly simulated by the numerical model fall in the range of the grid size to the domain size of the model. Although the LES of turbulent flows and neutral and unstable *planetary boundary layer* (PBL) flows have been demonstrated to be very encouraging, the simulations of the nocturnal boundary layer are less successful due to the fact that the characteristic large-eddy scale becomes

too small, and that most of the energy transfer and other exchange processes are overly influenced or dominated by subgrid scale motions. Although the LES derive their credibility from the explicit resolution of large-scale turbulent eddies, they depend upon a small-scale *turbulence closure* and must, to some degree, inherit the many uncertainties associated with turbulence closure (Mason 1994). Most LES results obtained so far are very encouraging, however, there is still room for improvements to overcome certain limiations. Some improvements include (a) the quality of the simulation can depend sensitively on subgrid modeling, which is not fully developed; and (b) LES requires high numerical accuracy, and does not in particular tolerate numerical dissipation which is often adopted in mesoscale models. To take advantage of both the LES and RANS, a hybrid LES-RANS approach has been developed and applied, in particular, to engineering problems.

Unresolved turbulent eddies of various scales smaller than the grid interval often fluctuate rapidly in time, thus limiting the description of their behavior to statistical approaches. The use of statistical approaches requires the introduction of an averaging operator. Any *averaging operator* used in atmospheric modeling should be able to satisfy the following criteria (Cotton 1986): (a) The operator should provide a formal mechanism for distinguishing between resolvable and unresolvable eddies; (b) The operator should produce a set of equations more amenable to integration, either analytically or numerically, than the original system of equations; and (c) The averaged set of atmospheric variables should be measurable by current or anticipated atmospheric sensing systems. Following the scheme originally developed by Reynolds (1895), each model variable is decomposed into a slow-varying mean field part and a rapid-varying

turbulent part, such as $u = \bar{u} + u'$, $v = \bar{v} + v'$, $w = \bar{w} + w'$, $\theta = \bar{\theta} + \theta'$, $p = \bar{p} + p'$, and $\rho = \bar{\rho} + \rho'$. Some useful formulas for the *Reynolds averaging* can be derived, for example,

$$\overline{u + w} = \bar{u} + \bar{w};\ \overline{cw} = c\bar{w};\quad \bar{\bar{w}} = \bar{w};\ \overline{w'} = 0;$$

$$\overline{w'\bar{\theta}} = \overline{w'}\,\bar{\theta} = 0;\ \overline{w\theta} = \overline{(\bar{w} + w')(\bar{\theta} + \theta')} = \bar{w}\,\bar{\theta} + \overline{w'\theta'}\ ;\ \overline{uw} = \bar{u}\,\bar{w} + \overline{u'w'};$$

$$\overline{\frac{\partial w}{\partial s}} = \frac{\partial \bar{w}}{\partial s};\ \frac{\overline{\partial \bar{w}}}{\partial s} = \frac{\partial \bar{w}}{\partial s};\ \int \overline{w\,ds} = \int \bar{w}\,ds;\quad s = x, y, z, \text{or } t, \tag{14.1.1}$$

where *c* is a constant, $\overline{u'w'}$ and $\overline{w'\theta'}$ are the *vertical turbulent flux of zonal momentum* and *vertical turbulent heat flux*, respectively. In statistical terms, these fluxes, as an average of the product of deviation components, are also called *covariances*. Figure 14.1 shows a sketch of subgrid-scale vertical velocity (w') and potential temperature (θ') and the subgrid scale covariance $\overline{w'\theta'}$. As can be seen from the figure, the vertical heat flux associated with the resolvable dependent variables is approximately zero, i.e. $\bar{w}\,\bar{\theta} \cong 0$ because $\bar{w} = 0$. However, the covariance or the vertical turbulent heat flux, $\overline{w'\theta'}$, is not 0. Both grid value averages are assumed to be constant over Δx.

If we apply the *Reynolds averaging* to a time interval and a grid volume of a numerical model, then the Reynolds-averaged value of a variable ϕ represents,

$$\bar{\phi} \equiv \frac{1}{\Delta x\,\Delta y\,\Delta z\,\Delta t}\int_t^{t+\Delta t}\int_x^{x+\Delta x}\int_y^{y+\Delta y}\int_z^{z+\Delta z}\phi\ dz\,dy\,dx\,dt\,. \tag{14.1.2}$$

This is called *grid-volume averaging*. Thus, $\phi' = \phi - \bar{\phi}$ is the fluctuation or perturbation across the grid intervals, Δx, Δy, Δz, and time interval Δt from $\bar{\phi}$. Applying the

Reynolds averaging to the grid volume of the mesoscale model system of (13.5.6)-(13.5.14) with Boussinesq approximation leads to

$$\frac{\overline{D}\overline{u}}{Dt} = f\overline{v} - \frac{1}{\rho_o}\frac{\partial \overline{p}}{\partial x} - \frac{1}{\rho_o}\left[\frac{\partial\left(\rho_o\overline{u'u'}\right)}{\partial x} + \frac{\partial\left(\rho_o\overline{u'v'}\right)}{\partial y} + \frac{\partial\left(\rho_o\overline{u'w'}\right)}{\partial z}\right] + \nu\nabla^2\overline{u}, \qquad (14.1.3)$$

$$\frac{\overline{D}\overline{v}}{Dt} = -f\overline{u} - \frac{1}{\rho_o}\frac{\partial \overline{p}}{\partial y} - \frac{1}{\rho_o}\left[\frac{\partial\left(\rho_o\overline{u'v'}\right)}{\partial x} + \frac{\partial\left(\rho_o\overline{v'v'}\right)}{\partial y} + \frac{\partial\left(\rho_o\overline{v'w'}\right)}{\partial z}\right] + \nu\nabla^2\overline{v}, \qquad (14.1.4)$$

$$\frac{\overline{D}\overline{w}}{Dt} = -\frac{1}{\rho_o}\frac{\partial p_1}{\partial z} - g\frac{\rho_1}{\rho_o} - \frac{1}{\rho_o}\left[\frac{\partial\left(\rho_o\overline{u'w'}\right)}{\partial x} + \frac{\partial\left(\rho_o\overline{v'w'}\right)}{\partial y} + \frac{\partial\left(\rho_o\overline{w'w'}\right)}{\partial z}\right] + \nu\nabla^2\overline{w}, \qquad (14.1.5)$$

$$\frac{\overline{D}\overline{\theta}_v}{Dt} = \overline{S}_\theta - \frac{1}{\rho_o}\left[\frac{\partial\left(\rho_o\overline{u'\theta'}\right)}{\partial x} + \frac{\partial\left(\rho_o\overline{v'\theta'}\right)}{\partial y} + \frac{\partial\left(\rho_o\overline{w'\theta'}\right)}{\partial z}\right] + \kappa\nabla^2\overline{\theta}, \qquad (14.1.6)$$

$$\frac{\overline{D}\overline{\phi}}{Dt} = \overline{S}_\phi - \frac{1}{\rho_o}\left[\frac{\partial\left(\rho_o\overline{u'\phi'}\right)}{\partial x} + \frac{\partial\left(\rho_o\overline{v'\phi'}\right)}{\partial y} + \frac{\partial\left(\rho_o\overline{w'\phi'}\right)}{\partial z}\right] + \kappa\nabla^2\overline{\phi},$$

$$\phi = q_v,\ q_c,\ q_i,\ q_r,\ q_s, \text{ and } q_g, \qquad (14.1.7)$$

$$\nabla\cdot\left(\rho_o\overline{\boldsymbol{V}}\right) = 0, \quad \overline{\boldsymbol{V}} = (\overline{u},\overline{v},\overline{w}), \qquad (14.1.8)$$

$$\overline{p} = \overline{\rho}R\overline{T}, \qquad (14.1.9)$$

$$\overline{\theta}_v = \overline{T}_v\left(\frac{p_s}{\overline{p}}\right)^{R_d/c_p}, \qquad (14.1.10)$$

$$\overline{T_v} = \overline{T}(1+0.61\overline{q}_v), \qquad (14.1.11)$$

$$\overline{p} = p_o + p_1;\ \overline{\rho} = \rho_o + \rho_1;\ \frac{\partial p_o}{\partial z} = -\rho_o g;\ \overline{\phi} = \phi_o + \phi_1, \qquad (14.1.12)$$

where p_s is 1000 hPa, and

$$\frac{\overline{D}}{Dt} = \frac{\partial}{\partial t} + \overline{u}\frac{\partial}{\partial x} + \overline{v}\frac{\partial}{\partial y} + \overline{w}\frac{\partial}{\partial z}, \tag{14.1.13}$$

$$\phi_o \equiv \frac{1}{XY}\int_{-Y/2}^{Y/2}\int_{-X/2}^{X/2} \overline{\phi}\, dx\, dy\,. \tag{14.1.14}$$

In the above equation, ϕ_o is the layer average over the domain (X,Y), which is assumed to be large enough compared with the mesoscale phenomena concerned, to ensure a hydrostatic balance. Since $\phi_1 = \overline{\phi} - \phi_o$, ϕ_1 is the nonhydrostatic part of $\overline{\phi}$ and/or the perturbation from large-scale ϕ_o. The terms $\overline{u'\theta'}$, $\overline{v'\theta'}$, and $\overline{w'\theta'}$ are the *turbulent heat fluxes*, $\overline{u'w'}$ and $\overline{v'w'}$ the *vertical turbulent fluxes* of horizontal momentum, and $\overline{u'v'}$ the *meridional turbulent flux* of zonal momentum. In deriving the above equations, we have assumed $\left|\rho_1/\rho_o\right| << 1$. We have also partitioned $\overline{p}$ and $\overline{\rho}$ into hydrostatic (with subscript "*o*") and nonhydrostatic parts (with subscript "*1*"). Above the boundary layer, these flux terms are very small compared with other terms and may thus be neglected. In the boundary layer, the turbulent flux divergence terms are of the same order as the other terms in (14.1.3)-(14.1.8). Therefore, they cannot be simply dropped from the equation system. Note that the equation set (14.1.3)-(14.1.11) is not a closed system mathematically since in addition to the unknown mean variables, other flux terms are also present. In order to make the equation set closed, we need to represent or parameterize the turbulent flux terms and the source and sink terms using the mean variables. The need for parameterizations poses a *closure problem*, which is a challenging task in parameterizing the PBL processes, as well as for moist and radiative transfer processes. The horizontal derivatives of the turbulent flux terms are normally

associated with some horizontal inhomogeneities, such as cities and coastlines, which may be neglected over horizontally homogeneous regions.

In addition to the Reynolds averaging method, several different averaging methods may also be adopted. For example, for a data set measured discretely, an *ensemble averaging* may be adopted,

$$\bar{\phi}_e = \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} \phi(x_o, y_{o,} z_o, t_o) . \qquad (14.1.15)$$

If the turbulence is stationary and homogeneous, which is unlikely in the real world, then the above three averaging methods should give the same value. An alternative approach in averaging data set is to take the *grid-volume averaging*, as defined in (14.1.2). For cases where there are N data points to be averaged over a grid volume, one may take the *generalized ensemble averaging*,

$$\bar{\phi} = \frac{1}{TXYZ} \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} \int_{-Z/2}^{Z/2} \int_{-Y/2}^{Y/2} \int_{-X/2}^{X/2} \int_{-T/2}^{T/2} \phi(x', y', z', t') \, dt' \, dx' \, dy' \, dz' . \qquad (14.1.16)$$

In mesoscale models, physical processes such as the planetary boundary, moist and radiative transfer, land-surface interaction, and air-sea interaction processes need to be parameterized. In this chapter, we will limit our discussion to the introduction of basic principles and methods in parameterizing the first three processes. However, this does not mean that the importance of the parameterization of other physical processes will be neglected.

14.2 Parameterization of planetary boundary layer processes

The objective of the PBL parameterization is to use the grid-volume averaged mean variables to represent the turbulent flux terms associated with turbulent eddies, as well as

the heat source and sink terms present in the PBL, such as those present in (14.1.3)-(14.1.8). In other words, we need to close the system mathematically. Thus, a suitable closure scheme is needed. Using the appropriate parameterizations of these terms, we are able to numerically integrate the Reynolds-averaged equations of the mean motion, such as (14.1.3)-(14.1.11).

One simple way to model the planetary boundary layer is to treat the whole layer as one slab and predict the vertically averaged properties of the PBL. In this approach, the details of the vertical structure of the PBL are ignored, which may work for cases where vertical gradients are small throughout much of the PBL, such as the convective boundary layer, or for use in models such as general circulation models (GCMs), which do not have enough vertical resolution. However, this approach is not appropriate for mesoscale models since the prediction of some detailed information of the PBL is expected. In mesoscale models, the PBL is divided into a number of layers, depending upon what the concerned physical phenomenon requires.

Physically, the PBL may be approximately divided into the *surface layer* and the layer above it. In the lowest part of the surface layer, i.e., the *viscous sublayer*, molecular motions dominate the transfer of dependent variables. The viscous sublayer forces the velocity to vanish (i.e., no-slip boundary condition) at the ground, which continuously leads to the development of turbulent eddies. Thus, the molecular viscosity and thermal diffusion terms in these equations are kept, but the turbulent flux terms in (14.1.3) - (14.1.7) are neglected. The viscious sublayer normally has a depth of O(1 cm), but can be as shallow as 0.001 cm over smooth ice. Over most natural surfaces, a roughness layer or canopy layer forms above the viscous sublayer. The roughness layer may go up

to 10 m over large buildings (Oke 1987), and the height is referred to as *aerodynamic roughness*, usually denoted as z_o. Due to the constraint of vertical resolution, the viscous sublayer and roughness layer of the surface layer are often neglected in mesocale models.

Above the viscous sublayer or roughness layer exists the upper part of the surface layer, which usually occupies up to 10-100 m, about 10% of the entire PBL. The upper part of the surface layer, which is often referred to simply as the "surface layer," is mainly maintained by the vertical momentum transfer associated with turbulent eddies. Coriolis and pressure gradient forces do not play a major role in the surface layer. Therefore, for the purpose of modeling the surface layer, it is normally assumed that: (1) the subgrid scale fluxes independent of z, (2) the Coriolis force negligible, (3) a steady state, and (4) there is a horizontal homogeneity over a flat surface. Based on these assumptions, empirical formulas have been developed to specify the relationship between dependent variables and subgrid fluxes. The layer above the surface layer becomes the *mixed layer* under unstable and convective conditions and the *outer layer* under neutral and stable conditions.

The mixed layer and outer layer have very different characteristics in terms of wind, temperature, and humidity profiles. Figure 14.2 shows typical profiles of mean virtual potential temperature, specific humidity, wind speed, and heat, moisture, and momentum fluxes within a convective boundary layer. The virtual potential temperature and wind speed are quite uniform in the mixed layer due to mixing associated with turbulent eddies. On top of the mixed layer is the *transition layer*, which contains a temperature inversion and an increase in wind speed, and then the *free atmosphere*. On the other

hand, a typical *stable boundary layer*, say induced by the nocturnal boundary layer, has an inversion layer above the surface (Fig. 14.3). Within the stable boundary layer, the atmosphere has its largest stability near the surface, decreasing smoothly toward neutral with height. A temperature inversion is often observed near the surface. Higher in the stable boundary layer, the wind speed may increase with height, reaching a maximum near the top of the stable layer, and thus becoming what is known as the *nocturnal low-level jet*. The whole depth of the stable boundary layer is on the order of several hundred meters, much shallower than the convective boundary layer.

14.2.1 Parameterization of the surface layer

Simulating the surface layer is important since momentum generated in the free atmosphere and the PBL tends to dissipate in this layer, while the heat and moisture are transported upward to the PBL from the ground through this layer. Analogous to molecular diffusion, the subgrid scale fluxes may be represented by

$$\overline{u'w'} = -K_m \frac{\partial \bar{u}}{\partial z}; \quad \overline{v'w'} = -K_m \frac{\partial \bar{v}}{\partial z}; \quad \overline{w'\theta'} = -K_h \frac{\partial \bar{\theta}}{\partial z}; \quad \overline{w'q'} = -K_q \frac{\partial \bar{q}}{\partial z}, \qquad (14.2.1)$$

where K_m is called the *exchange coefficient of momentum* or simply *eddy viscosity*, and K_h, and K_q are called the *exchange coefficients* or *eddy diffusivities of heat and water vapor*, respectively. In practice, K_q is often assigned as the same value as K_h. The exchange coefficients are often taken as constants, and are empirically related to height and stability as calculated from the NWP model output, and this approach of the parameterization of momentum, heat, and moisture fluxes is called *K theory*. The K theory is a *first-order closure* since the fluxes are parameterized proportional to the mean

values, which have also been applied to the parameterization of the PBL above the surface layer. Since the subgrid fluxes are assumed to be independent of height in the surface layer, we may define a *friction velocity* u_*, *flux temperature* θ_*, and q_* as

$$u_*^2 = \tau_o / \overline{\rho} = \left(\overline{u'w'}^2 + \overline{v'w'}^2 \right)^{1/2};$$

$$u_*^2 \cos\mu = -\overline{u'w'}; \quad u_*^2 \sin\mu = -\overline{v'w'}; \; \mu = \tan^{-1}(\overline{v}/\overline{u});$$

$$u_*\theta_* = -\overline{w'\theta'}; \;\; u_* q_* = -\overline{w'q'}, \tag{14.2.2}$$

where τ_o is the shearing stress generated by the horizontal wind. If the *x*-axis is chosen such that $\overline{v} = \overline{u'v'} = 0$, then (14.2.2) reduces to $u_*^2 = -\overline{u'w'}$. From dimensional analysis, the wind shear ($\partial\overline{V}/\partial z$) is proportional to the velocity scale (u_*) divided by the length scale (*z*),

$$\frac{\partial \overline{V}}{\partial z} = \frac{u_*}{kz}; \quad \overline{V} = \sqrt{\overline{u}^2 + \overline{v}^2}, \tag{14.2.3}$$

where *k* is the *von Kármán constant*, which has an empirical value of 0.4. Integrating (14.2.3) leads to the well-known logarithmic wind profile for a neutrally-stratified, constant-flux surface layer,

$$\overline{V}(z) = \frac{u_*}{kz} \ln\left(\frac{z}{z_o} \right). \tag{14.2.4}$$

Based on the Monin and Obukov similarity theory, we have

$$\frac{\partial \overline{V}}{\partial z} = \frac{u_*}{kz} \phi_m(z/L), \tag{14.2.5}$$

where L is the *Monin-Obukov length* defined as $L = \overline{\theta} u_*^2 / kg\theta_*$, $\theta_* = -\overline{w'\theta'}/u_*$, and ϕ_m is estimated by an empirical formula, such as (Businger 1973),

$$\phi_m \approx (1-15z/L)^{-1/4}, \quad z/L \le 0 \tag{14.2.6}$$

$$\approx 1+4.7z/L, \qquad z/L > 0 .$$

Integrating (14.2.5) from z_o, where $\overline{V} = 0$, to z gives

$$\overline{V}(z) = \frac{u_*}{k}\left[\ln\left(\frac{z}{z_o}\right) - \varphi_m\left(\frac{z}{L}\right)\right], \tag{14.2.7}$$

where

$$\varphi_m(z/L) = 2\ln\left((1+\phi_m^{-1})/2\right) + \ln\left((1+\phi_m^{-2})/2\right) - 2\tan^{-1}(1/\phi_m) + \pi/2, \quad z/L \le 0$$

$$= -4.7z/L, \qquad z/L > 0 \tag{14.2.8}$$

Similarly, the vertical profiles of $\overline{\theta}$ and $\overline{q}$ can be formulated as

$$\overline{\theta}(z) = \overline{\theta}_{z_o} + \frac{\theta_*}{\beta k}\left[\ln\left(\frac{z}{z_o}\right) - \varphi_h\left(\frac{z}{L}\right)\right], \tag{14.2.9}$$

$$\overline{q}(z) = \overline{q}_{z_o} + \frac{q_*}{\beta k}\left[\ln\left(\frac{z}{z_o}\right) - \varphi_h\left(\frac{z}{L}\right)\right], \tag{14.2.10}$$

where φ_h is an empirical formula. For example, the following φ_h has been proposed (Businger 1973),

$$\varphi_h(z/L) = \begin{cases} 2\ln\left[(1+0.74\phi_h^{-1})/2\right], & z/L \le 0 \\ -6.35z/L, & z/L > 0 \end{cases} \tag{14.2.11}$$

$$\phi_h = \begin{cases} 0.74(1-9z/L)^{-1/2}, & z/L \le 0 \\ 0.74+4.7z/L, & z/L > 0 \end{cases} \tag{14.2.12}$$

The symbol β represents the characteristic vertical mixing length for $\overline{\theta}$ and $\overline{q}$. An empirical value of 1.35 has been used. Note that at the bottom of the surface layer (i.e. $z = z_o$) or the top of the *viscous sublayer* or roughness layer, the no free-slip conditions for the velocities are often assumed, i.e. $\overline{u} = \overline{v} = \overline{w} = 0$, and the potential temperature and specific humidity may be estimated by (Deardorff 1974)

$$\overline{\theta}_{z_o} = \overline{\theta}_G + 0.0962\,(\theta_* / k)(u_* z_o / \nu)^{0.45} \text{ and} \tag{14.2.13}$$

$$\overline{q}_{z_o} = \overline{q}_G + 0.0962\,(q_* / k)(u_* z_o / \nu)^{0.45}, \tag{14.2.14}$$

where ν is the kinematic viscosity coefficient of air, which has a value of about $1.5\text{x}10^{-5} m^2 s^{-1}$.

14.2.2 Parameterization of the PBL

As mentioned above, the layer above the surface layer is called the *mixed layer* under unstable conditions, and the *outer layer* under neutral and stable conditions. The mixed layer extends from the top of the surface layer to 1-2 km or higher under unstable conditions, and several hundred meters under neutral and stable conditions. Three boundary layer flow regimes, based on different sets of force balances, have been proposed to help understand the dynamics of the PBL above the surface layer: (1) *Ekman layer*, which is supported by a balance among the pressure gradient force, Coriolis force, and frictional force, (2) *advective boundary layer*, which is supported by a balance among pressure gradient force, frictional force, and advective acceleration, and (3) *Stokes boundary layer*, which is supported by a balance between pressure gradient force and frictional force. In order to close the equation set (14.1.3)-(14.1.11), the subgrid scale

fluxes need to be represented by the mean values averaged over grid and time intervals. From a numerical modeling point of view, the parameterizations of the PBL above the surface layer may be roughly classified as: (a) bulk aerodynamic parameterization , (b) K-theory parameterization, (c) turbulent kinetic energy closure scheme, and (d) higher-order closure schemes.

a. Bulk aerodynamic parameterization

The *bulk aerodynamic parameterization* treats the boundary layer as a single slab and assumes the wind speed and potential temperature are independent of height, and the turbulence is horizontally homogeneous. Based on these assumptions, the horizontal turbulence flux divergence terms in (14.1.3)-(14.1.7) can be neglected, and the vertical subgrid turbulence fluxes are parameterized by

$$\overline{u'w'} = -C_d \overline{V}^2 \cos\mu\,;\ \overline{v'w'} = -C_d \overline{V}^2 \sin\mu\,;\ \overline{w'\theta'} = -C_h \overline{V}\left(\overline{\theta} - \overline{\theta}_{z_o}\right), \qquad (14.2.15)$$

where C_d and C_h are nondimensional *drag* and *heat transfer coefficients*, respectively, $\overline{V} = (\overline{u}^2 + \overline{v}^2)^{1/2}$, $\mu = \tan^{-1}(\overline{v}/\overline{u})$, and z_o is the roughness or top of the surface layer. The values of $\overline{V}$, $\overline{u}$, $\overline{v}$, and $\overline{\theta}$ are evaluated at the standard anemometer height, 10 m. The bulk aerodynamic parameterization has been adopted in some GCM and regional climate models. For a given reference height, C_d increases with increasing roughness, which ranges from $1.3\text{x}10^{-3}$ over ocean surface to $7\text{x}10^{-3}$ over rough land surface. From the formulas proposed in the parameterization of the surface layer, the expressions of C_d and C_h may be derived

$$C_d = k^2 / \left(\ln(z/z_o) - \varphi_m(z/L)\right)^2 ;$$

$$C_h = \beta k^2 / \left(\ln(z/z_o) - \varphi_m(z/L)\right)\left(\ln(z/z_o) - \varphi_h(z/L)\right), \qquad (14.2.16)$$

where φ_m and φ_h are defined in (14.2.8) and (14.2.12), respectively, and β is an empirical value defined in (14.2.9). As mentioned earlier, an empirical value 1.35 has been proposed for β.

Due to the assumption of height-independent wind speed and potential temperature and horizontally homogeneous turbulence, the bulk aerodynamic parameterization is more suitable for representing a well-mixed boundary layer than the neutral and stable boundary layers. Based on these assumptions, further assuming a three-way balance among the Coriolis force, pressure gradient force, and the vertical gradient of the turbulent momentum flux from (14.1.3) and (14.1.4), and using the bulk parameterization, one may derive the following equations for $\overline{u}$ and $\overline{v}$,

$$\overline{u} = \overline{u}_g - \kappa_s \overline{V}\overline{v} \; ; \; \overline{v} = \kappa_s \overline{V}\overline{u} \, , \qquad (14.2.17)$$

where $\kappa_s \equiv C_d / (fh)$, h is the mixed layer height, and $\overline{u}_g$ is the geostrophic wind speed at the bottom of the mixed layer. Equation (14.2.17) can also be rewritten as

$$f\mathbf{k} \times \overline{\mathbf{V}} = -\frac{1}{\rho_o}\nabla \overline{p} - \frac{C_d}{h}\overline{V}\overline{\mathbf{V}}; \quad \overline{\mathbf{V}} = (\overline{u}, \overline{v}), \qquad (14.2.18)$$

which gives a three-way balance with the wind deflected toward the low pressure. In addition, the cross-isobar flow increases as the turbulent drag increases. Note that in a rotational frame of reference or in the presence of directional shear, the frictional force on a fluid element need not be parallel and opposite to the velocity vector (e.g., Fig. 6.4

of Arya 2001), as commonly depicted in many textbook schematics of the force balance in the frictional layer (e.g., Holton 2004).

b. K-theory parameterization

Although the bulk parameterization is simple and easy to implement in a numerical model, it cannot properly represent a neutrally and stably stratified boundary layer. The reason for this is that the wind speed and direction in this situation does vary significantly with height and the boundary layer above the surface cannot be treated as a single slab. In order to close the mathematical problem, the subgrid turbulent flux terms are assumed to be proportional to their corresponding local gradients of the mean values, analogous to molecular diffusion. In this approach, the turbulent flux terms in (14.1.3)-(14.1.7) are written as (14.2.1).

Similar to the bulk parameterization, the subgrid turbulent flux divergence terms are neglected. The simplest way to determine the exchange coefficients in the boundary layer is based on the *mixing length hypothesis*. Analogous to the mean free path of molecules, the mixing length hypothesis assumes that an air parcel that is displaced vertically will carry the mean properties of its original level for a characteristic length, i.e. the *mixing length* (l), before mixing with its environment. Since $lu' \approx \partial \bar{u} / \partial z$ and K_m are proportional to lu', based on dimensional argument, we then have

$$K_m = l^2 \, \partial \bar{u} / \partial z \, . \qquad (14.2.19)$$

The eddy and thermal diffusivity coefficients, K_m and K_h, respectively, are often taken as either constants or empirically related to height and stability as calculated from NWP model output. As mentioned in the parameterization of the surface layer discussion, this

approach of the parameterization of momentum, heat, and moisture fluxes is referred to as *K theory*. The K theory is a *first-order closure* because the fluxes are parameterized proportional to the mean values. If the exchange coefficients are taken as constants, then they are referred to as *local exchange coefficients*. For example, the local exchange coefficient may be expressed as (Blackadar 1979)

Stably stratified ($\partial\overline{\theta}/\partial z > 0$):

$$K_m = K_h = 1.1(Ri_C - Ri)l^2(\partial\overline{V}/\partial z)/Ri_C, \; Ri \le Ri_C \qquad (14.2.20a)$$

$$= 0, \qquad Ri > Ri_C,$$

Unstably stratified ($\partial\overline{\theta}/\partial z \le 0$):

$$K_m = l^2(\partial\overline{V}/\partial z)(1-21Ri)^{1/2}; \; K_h = l^2(\partial\overline{V}/\partial z)(1-87Ri)^{1/2}, \qquad (14.2.20b)$$

where $Ri_C = 0.25$ is the critical Richardson number. Note that Ri_C distinguishes whether the flow is dynamically (shear) stable or not. A value of $l = kz$ for z < 200 m (with $k = 0.35$) and 700 m for $z \ge 200$ m in (14.2.20a) has been suggested (McNider and Pielke 1981).

The *local K-theory* approach has been adopted in a number of mesoscale models as an option. In addition to Blackadar's formulation, other formulations of local exchange coefficients have also been proposed. However, approaches such as the local K-theory scheme have been found to have some deficiencies. The most serious problem in this formulation is that the transport of mass and momentum in the PBL is mostly accomplished by the largest eddies and such eddies should be parameterized by the bulk properties of the PBL instead of the local properties (e.g., Wyngaard and Brost 1984;

Holtslag and Moeng 1991). The discrepancy in eddy size makes the local K-theory problematic for unstable conditions, and its implementation could induce the appearance of countergradient fluxes. In order to resolve this problem, *non-local K-theory* has been proposed (e.g., Deardorff 1972; Troen and Mahrt 1986; Holtslag and Moeng 1991). For example, the turbulence diffusion equations for prognostic variables can be expressed by

$$\frac{\partial \phi}{\partial t} = \frac{\partial}{\partial z}\left[K_c \left(\frac{\partial \phi}{\partial z} - \gamma_C \right) \right], \quad \phi = u,\, v,\, w,\, \theta, \text{or } q \tag{14.2.21}$$

where $K_c = K_m$ or K_h and γ_C is a correction to the local gradient that incorporates the contribution of the large-scale eddies to the total flux. The eddy diffusivity coefficient can be formulated as

$$K_m = k w_s z \left(1 - \frac{z}{h} \right)^p , \tag{14.2.22}$$

where p is the profile shape exponent taken to be 2, k is the von Karman constant (= 0.4), z is the height from the surface, h is the height of PBL, and w_s is a mixed-layer velocity scale (e.g., Troen and Mahrt 1986; Hong and Pan 1996).

Assuming a three-way balance among the Coriolis force, pressure gradient force, and the vertical gradient of the turbulent momentum flux from (14.1.3) and (14.1.4), in addition to the use of the K-theory parameterization with constant K_m, one may derive the following *Ekman layer* relationships

$$K_m \frac{\partial^2 \overline{u}}{\partial z^2} + f(\overline{v} - \overline{v}_g) = 0 , \tag{14.2.23}$$

$$K_m \frac{\partial^2 \overline{v}}{\partial z^2} - f(\overline{u} - \overline{u}_g) = 0 . \tag{14.2.24}$$

The derivation of the above equations is similar to that of (14.2.17), except that the K-theory parameterization is adopted instead of the bulk parameterization. Introducing a new complex variable, $u + iv$, (14.2.23), and (14.2.24) can be combined into a single equation,

$$K_m \frac{\partial^2(\overline{u} + i\overline{v})}{\partial z^2} - if(\overline{u} + i\overline{v}) = -if(\overline{u}_g + i\overline{v}_g). \tag{14.2.25}$$

The solution of (14.2.25) subjected to the no-slip boundary conditions at the ground, $\overline{u} = \overline{v} = 0$ at $z = 0$, and approaching geostrophic wind speeds far from the ground, i.e. $\overline{u} \to \overline{u}_g$ and $\overline{v} \to \overline{v}_g$ as $z \to \infty$ is

$$\overline{u} = \overline{u}_g(1 - e^{-\gamma z}\cos\gamma z);\ \overline{v} = \overline{u}_g e^{-\gamma z}\sin\gamma z, \tag{14.2.26}$$

where $\gamma = (f/2K_m)^{1/2}$. The above solution is sketched in Fig. 14.4. The wind veers (i.e. turns clockwise) and increases with height to be slightly over the geostrophic value, and then reaches to be nearly the geostrophic value at $z = \pi/\gamma$, which may also be defined as the Ekman layer depth. The spiral wind profile is known as *Ekman spiral*.

c. Turbulent kinetic energy closure scheme

The first-order closure schemes, such as K-theory parameterization, may be improved by predicting one of the subgrid-scale variables, the turbulent kinetic energy (TKE) per unit mass $\left[e = (\overline{u'^2} + \overline{v'^2} + \overline{w'^2})/2\right]$, while the other subgrid scale turbulent flux terms are diagnosed and related to both the TKE and the grid-scale mean values. The prognostic prediction of TKE in the parameterization scheme is referred to as the *TKE* or *one-and-a-half-order closure*.

When the Reynolds number of a laminar flow increases, it may break down into a turbulent flow. A turbulent flow is characterized by high randomness, nonlinearity, diffusivity, vorticity, and dissipation. The breakdown is often associated with instability, such as shear instability or buoyant (static) instability. Shear and buoyancy are the two major sources of the production of TKE, which may be denoted as *S* and *B*, respectively. Once the turbulence is generated and fully developed into a steady state in terms of averaged flow properties, then the instability is no longer required to sustain the turbulent flow. In order to reach steady state turbulence (statistically), certain mechanisms are required to remove and redistribute TKE. These mechanisms are often attributed to the dissipation (*D*) due to turbulent eddy viscosity and molecular viscosity, and the transport and redistribution (Tr) due to advection and pressure forces. Thus, the time evolution of TKE can be written as

$$De/Dt = S + B + Tr - D\,. \tag{14.2.27}$$

To derive the mathematical form of the TKE equation, we first substitute $u = \overline{u} + u'$, $v = \overline{v} + v'$, $w = \overline{w} + w'$, $p = \overline{p} + p' = p_o + p_1 + p'$, $\theta = \overline{\theta} + \theta' = \theta_o + \theta_1 + \theta'$, $\rho = \overline{\rho} + \rho' = \rho_o + \rho_1 + \rho'$ into (13.5.6)-(13.5.8) with *f* neglected, to obtain,

$$\frac{D(\overline{u} + u')}{Dt} = -\frac{1}{\rho_o}\frac{\partial(\overline{p} + p')}{\partial x} + \nu\nabla^2(\overline{u} + u')\,, \tag{14.2.28}$$

$$\frac{D(\overline{v} + v')}{Dt} = -\frac{1}{\rho_o}\frac{\partial(\overline{p} + p')}{\partial y} + \nu\nabla^2(\overline{v} + v')\,, \tag{14.2.29}$$

$$\frac{D(\overline{w} + w')}{Dt} = -\frac{1}{\rho_o}\frac{\partial(p_1 + p')}{\partial z} - \frac{g}{\rho_o}(\rho_1 + \rho') + \nu\nabla^2(\overline{w} + w')\,. \tag{14.2.30}$$

Unlike in Section 14.1, in deriving the above equation, we have used the partition of $\overline{p}$ and $\overline{\rho}$ into hydrostatic (p_o and ρ_o – large scale) and nonhydrostatic (p_1 and ρ_1 – mesoscale) parts, neglected ρ_1 and ρ' relative to ρ_o except in the buoyancy (associated with the gravity) term, and assumed an anelastic fluid. Now, multiplying (14.2.28)-(14.2.30) by u', v', w', respectively, and then taking the Reynolds averaging over a grid volume lead to the *TKE equation*,

$$\frac{\partial \overline{e}}{\partial t} = -\overline{\boldsymbol{V}} \cdot \nabla \overline{e} - \overline{\boldsymbol{V}' \cdot \nabla e} - \left(\frac{1}{\rho_o}\right)\left[(\overline{u'p'})_x + (\overline{v'p'})_y + (\overline{w'p'})_z\right] - \left(\frac{g}{\rho_o}\right)\overline{\rho' w'}$$

$$\text{1} \qquad \text{2} \qquad\qquad\qquad\qquad \text{3} \qquad\qquad\qquad\qquad \text{4}$$

$$-\left[\left(\overline{u'u'}\,\overline{u}_x + \overline{u'v'}\,\overline{u}_y + \overline{u'w'}\,\overline{u}_z\right) + \left(\overline{u'v'}\,\overline{v}_x + \overline{v'v'}\,\overline{v}_y + \overline{v'w'}\,\overline{v}_z\right)\right. \qquad \text{, (14.2.31)}$$

$$\text{5}$$

$$\left. + \left(\overline{u'w'}\,\overline{w}_x + \overline{v'w'}\,\overline{w}_y + \overline{w'w'}\,\overline{w}_z\right)\right] + \nu\nabla^2\overline{e} - \nu\left(\overline{u'^2_x} + \overline{v'^2_y} + \overline{w'^2_z}\right)$$

$$\text{6} \qquad\qquad \text{7}$$

The left-hand side of (14.2.31) represents the local rate of change of the TKE. Term 1 is the advection of $\overline{e}$ by the grid-volume averaged velocity. Term 2 represents the grid-volume average of the advection of TKE by the subgrid-scale perturbation velocity. Term 3 represents the change in TKE by advection through the boundaries of the grid volume. Term 3 is difficult to measure and is thus often ignored in the closure problem. Term 4 represents the buoyancy production of the TKE, while Term 5 represents the shear production of the TKE. Term 6 represents the diffusion of turbulence by molecular diffusion. Term 7 is the sink of TKE by molecular diffusion. In mesoscale modeling, Terms 6 and 7 are often ignored.

d. Higher-order closure schemes

In fact, subgrid-scale perturbations such as u', v', w', and θ', can be predicted by subtracting the resolved flow equations from the full equations, similar to the derivation of TKE equation. The proposed method will generate new unknown variables involving triple correlation of the perturbations, which must be represented by the mean variables and quadratic perturbation terms, in order to close the system mathematically. One can go further by deriving the prediction equations for the third moments and close the system on higher-order correlation terms (Mellor and Yamada 1974), commonly referred to as the *higher-order closures*. The higher-order closure schemes are capable of representing a well-mixed layer structure. Figure 14.5 shows a comparison of numerically simulated virtual potential temperature profiles in the boundary layer for Day 33 of the Wangara experiment by using a TKE closure scheme and a third-order closure scheme, and the observational data. The TKE closure scheme (Fig. 14.5a) is capable of capturing the observed major features (Fig. 14.5c) compared to the third-order closure scheme (Fig. 14.5b). The higher-order closure schemes are computationally expensive and do not necessarily make a significant improvement in accurately parameterizing the PBL compared to lower-order closure schemes, such as TKE.

14.3 Parameterization of moist processes

As discussed in Chapters 8 and 9, many severe mesoscale weather phenomena, such as thunderstorms, squall lines, mesoscale convective systems, rainbands, and frontal circulations, are associated with moist processes. In addition, the presence of water vapor and clouds in the atmosphere also play important roles in the reflection, absorption,

and emission of both solar and terrestrial radiation. In most mesoscale NWP models, the majority of clouds, especially convective clouds, cannot be resolved by the grid mesh system. Thus, the moist variables are parameterized by the grid-volume mean variables in a way analogous to the parameterization of turbulent eddies in the PBL. The situation is different in cloud models because normally the horizontal resolution is fine enough to roughly represent the clouds. The microphysical processes, however, still need to be parameterized or appropriately represented. Thus, the accurate representation of the moist processes in mesoscale NWP and cloud models has become one of the most challenging tasks in mesoscale modeling. Many details of the parameterization schemes are still topics of current research. Thus, in this section, it is only appropriate to make a brief summary of the representation or parameterization of these moist processes in mesoscale NWP models. Detailed discussions on individual parameterization schemes in microphysical and cumulus processes can be found in relevant literature.

As discussed in Chapter 13, a dry atmospheric system can be described by the horizontal and vertical momentum equations, the continuity equation, the thermodynamic equation, and the equation of state, which is composed of six equations with six unknown variables. If potential temperature is adopted in the thermodynamic equation, then the Poisson equation is needed to close the system. For example, (13.5.6)-(13.5.9) and (13.5.11)-(13.5.13) with q_v set to zero describes this dry atmospheric system. When a moist atmospheric system is considered, however, effects of moist processes have to be represented in the heat source or sink term (S_θ) of the thermodynamic equation (13.5.9), and additional equations for the hydrometeors are needed to describe the moist processes. Equations (13.5.6)-(13.5.14) describe this type of moist atmospheric system. The

derivation of equations for hydrometeors is similar to that of the continuity equation for dry air, based on the conservation of mass. Different types of clouds are described by a different number of equations for hydrometeor effects. For example, for warm clouds, three additional equations for the water vapor, cloud water, and rainwater are required, while for cold clouds we need to add equations for cloud ice, snow, and graupel or hail. Various source and sink terms portraying different hydrometeors are also represented by the S_ϕ term in (13.5.10). In addition, virtual temperature and virtual potential temperature, instead of temperature and potential temperature, are often used in the equation system because q_v is no longer zero.

The treatments of moist processes in mesoscale models may be divided into two categories: (1) *parameterization of microphysical processes*, and (2) *cumulus parameterization*. In the first category, the microphysical processes are represented by the continuity equations for each hydrometeor, such as those described in (13.5.10). The source terms on the right-hand side of (13.5.10) must be formulated in a way that all possible microphysical interactions among different categories of hydrometeors are included. Two approaches have been taken: (a) *explicit representation*, and (b) *bulk microphysics parameterization* (*BMP*).

14.3.1 Parameterization of microphysical processes

a. Explicit representation

In the explicit representation of the microphysical processes, each category of the hydrometeors, such as water vapor, cloud water, cloud ice, rain, snow, and graupel/hail, is represented by a continuity equation, based on the conservation of mass. Each

hydrometeor is further divided into different subcategories, based on the size. For example, the liquid water mixing ratio in a warm cloud may be approximated by

$$q_i = \frac{1}{\rho}\int_0^\infty mN(m)dm \approx \frac{1}{\rho}\sum_{i=1}^{k} m_i N_i \Delta m_i \,, \tag{14.3.1}$$

where ρ is the air density, m is the cloud water mass, $N(m)$ is the size distribution for cloud water, $mN(m)$ is the total number of cloud droplets in mass range m to $m+dm$ per unit volume of air, and subscript i denotes a subcategory. The continuity equation for liquid water may then be written as

$$\frac{DN_i}{Dt} = -N_i \nabla \cdot \boldsymbol{V} + P_{AUTO} + P_{DIFF} + P_{ACCR} + P_{BREK} + P_{FALL} \,, \tag{14.3.2}$$

where the P terms represent microphysical processes responsible for production or reduction of cloud water in a warm cloud, which include condensation from water vapor (P_{AUTO}), vapor diffusion (condensation or evaporation) (P_{DIFF}), accretion (P_{ACCR}), drop breakup (P_{BREK}), and fallout (P_{FALL}).

The continuity equation for water vapor may be written as

$$\frac{Dq_v}{Dt} = -\frac{1}{\rho}\sum_{i=1}^{k} m_i (P_{AUTO} + P_{DIFF})\Delta m_i + \kappa \nabla^2 q_v \,, \tag{14.3.3}$$

Depending upon the number of size categories adopted, it may easily exceed 50 equations for cloud water alone (see (14.3.2)). In addition, the interactions among different size categories become very complicated and the calculations become very tedious and computationally expensive.

b. Bulk microphysics parameterization

Instead of explicit representation of microphysical processes, an alternative is to perform the *bulk microphysics parameterization* (BMP). In taking the bulk parameterization approach, each category of the hydrometeor is governed by its own continuity equation, based on the conservation of mass. In order to avoid calculations of complicated interactions among the different sizes of hydrometeor particles, the shape and size distributions are often assumed a priori and the basic microphysical processes are parameterized. The concept of bulk microphysical parameterization can be understood by considering a nonprecipitating cloud which contains only water vapor (q_v) and cloud water (q_c). Since the total water-substance mixing ratio, $q_T = q_v + q_c$, is conserved, the continuity equations for water vapor and cloud water are,

$$\frac{Dq_v}{Dt} = -c, \tag{14.3.4}$$

$$\frac{Dq_c}{Dt} = c, \tag{14.3.5}$$

where c represents the condensation rate of water vapor into cloud water when $c > 0$ and evaporation rate when $c < 0$. The rewritten contunity equations can be extended to the *warm clouds with precipitation*, which include water vapor, cloud water, and rain, such as the *warm-rain BMP* scheme proposed by (Kessler 1969). In the warm-rain bulk parameterization, more source and sink terms associated with microphysical processes, such as evaporation of cloud water, evaporation of rainwater, autoconversion of cloud water to form rain, and accretion of cloud water by rainwater, need to be added to the right hand side of the above equations. In addition, a continuity equation for rainwater with a fallout term should also be added to the system.

The warm rain BMP has been extended to include ice phase components such as cloud ice, snow, and graupel/hail (e.g., Lin, Farley, and Orville 1983 - LFO scheme). In this BMP scheme, rain, snow, and graupel are assumed to have *terminal velocities* in order to precipitate downward, while the rest do not possess terminal velocities. Figure 14.6 illustrates the cloud microphysical processes in the LFO scheme. The symbols are explained in Table 14.1. The major microphysical processes include *autoconversion* (growth from only one category of hydrometeors, such as condensation and aggregation), *evaporation*, *sublimation*, *freezing*, *melting*, *accretion* (growth between different categories of hydrometeors), *Bergeron process* (growth of ice at the expense of cloud water in cold clouds because the satuaration vapor pressure with respect to ice is less than that with respect to water), and *dry* and *wet growth* of graupel.

In the scheme sketched in Fig. 14.6, the shape of liquid water and ice particles are assumed to be spherical. The *size distributions* of precipitation particles, i.e., rain (q_r), snow (q_s), and graupel or hail (q_g), are hypothesized as

$$N_k(D) = N_{ok} \exp(-\lambda_k D_k), \quad k = r,\ s,\ \text{or g} \tag{14.3.6}$$

where N_{ok} and λ_k are the *intercept* and *slope parameters* of the size distribution, respectively, and D_k is the diameter of the hydrometeor. Equation (14.3.6) is called the *Marshall-Palmer* (1948) *distribution*. The slope parameter λ_k is determined by multiplying (14.3.6) by the particle mass and integrating over all diameters and equating the resulting quantities to the appropriate water contents, which leads to

$$\lambda_k = \left(\frac{\pi \rho_k N_{ok}}{\rho q_k} \right)^{0.25} . \tag{14.3.7}$$

Since N_{ok} is held constant and only λ_k is prognostic, this type of BMP scheme is called *one-moment BMP scheme*.

The continuity equations for the water vapor and five categories of hydrometeors may be written as

$$\frac{\partial q_j}{\partial t} = -\boldsymbol{V} \cdot \nabla q_j + \nabla \cdot K_h \nabla q_j + P_j , \quad j = v,\ c,\ \text{or}\ i , \tag{14.3.8}$$

$$\frac{\partial q_k}{\partial t} = -\boldsymbol{V} \cdot \nabla q_k + \nabla \cdot K_m \nabla q_k + P_k + \frac{1}{\rho}\frac{\partial}{\partial z}(\rho U_k q_k) , \quad k = r,\ s,\ \text{or}\ g , \tag{14.3.9}$$

where U_k represents the terminal velocities of precipitation hydrometeors (rain, snow, and graupel), K_m and K_h are the eddy viscosity and eddy thermal diffusivity, respectively, where the last term of (14.3.9) is the *fallout term*. The production (i.e. *P* terms, see Table 14.1) terms are sketched in Fig. 14.6. Note that the subgrid scale flux terms have been parameterized by the *K*-theory closure.

Figure 14.7 shows simulations of tropical cyclones using the LFO scheme and the importance of ice phase in the simulations. The warm-rain simulation shows an outward-sloping eyewall, subsidence inside the eyewall, and an area of mesoscale ascent extending 20-30 km out from the eyewall (Fig. 14.7a). The ice-phase simulation shows a similarly sloping eyewall below 5-6 km, but above this level the eyewall updrafts become more vertically oriented. An area of mesoscale ascent containing several convective updrafts is located radially outward from the convective ring at 60-70 km. The downdrafts in the ice-phase simulation are stronger and more coherent horizontally (Fig. 14.7c). They tend to originate near the melting level (dashed line). The tangential wind of the warm-rain simulation indicates maximum winds at r = 33-35 km in a deeper layer

(Fig. 14.7b), while the ice-phase simulation shows maximum winds at r = 17-19 km in a shallow layer (Fig, 14.7d).

Different approaches have been taken to parameterize ice-phase microphysical processes, such as treating aggregates of ice crystals as a distinct snow species (e.g., Cotton et al. 1986), and to improve the microphysical parameterization schemes, such as the addition of collision between snow and cloud water (riming) (Rutledge and Hobbs 1984; Chen and Sun 2002; Lin et al. 2005). Other research has improved these schemes through the inclusion of four ice classes: small ice crystals, snow, graupel and frozen drops/hail (Ferrier 1994), ensuring that supersaturation (subsaturation) cannot exist at a grid point that is clear (cloudy) (Tao et al. 2003), and diagnosis of the cloud ice number concentration from its mixing ratio (Hong et al. 2004). In real-time NWP models, simplified BMP schemes have been developed to make numerical simulations more efficient computationally. The most significant improvement is the development of two- or multi-moment BMP schemes. In general, the size distribution (14.3.6) includes the shape factor and is written as

$$N_k(D) = N_{ok} D_k^{\alpha} \exp(-\lambda_k D_k), \quad k = r,\ s,\ \text{or g}, \tag{14.3.10}$$

where α is called the *shape parameter*. Thus, there are three parameters or moments, N_{ok}, λ_k, and α, to be determined. Following Kessler's (1969) warm-rain scheme, the LFO scheme ((14.3.6) and Fig. 14.6) assumes spherical precipitation particles ($\alpha = 0$) and that N_{ok} is a contatnt, which yields a one-moment scheme. If two of these parameters, such as N_{ok} and λ_k, are prognostic, and the third parameter (α) is held constant, the scheme is called two-moment scheme (e.g., Ferrier 1994; Meyers et al. 1997; Reisner et

al. 1998; Morrison and Pinto 2005; Seifert and Beheng 2006). If all of these three parameters are prognostic, then it is called three-moment scheme (e.g., Milbrandt and Yau 2005).

14.3.2 Cumulus parameterization

Even though most individual cumulus clouds have horizontal scales smaller than the mesoscale model grid mesh, the collective effects of cumulus clouds, such as the convective condensation and transport of heat, moisture, and momentum, on the larger scale environment are essential and need to be represented by grid-scale variables. On the other hand, the large-scale forcing tends to modulate the cumulus convection, which in turn determines the total rainfall rate. The representation of these processes is carried out by the *cumulus parameterization* (*CP*) schemes. To parameterize the interactions between cumulus clouds and their environment, we must determine the relationship between cumulus convection and its larger-scale environment. In practice, CP schemes may be divided into schemes for large-scale models and schemes for mesoscale models. However, it is rather difficult to make a clear cut decision on distinguishing these two types of models. The mesoscale models may refer to models having grid spacing in between 10 to 50 km and a time step of several minutes or less, while the large-scale models may refer to models having grid spacing larger than 50 km and a time step greater than several minutes. For models having grid spacing less than 10 km, BMP are often employed. Cumulus parameterization poses a challenging problem in atmospheric modeling and is still a topic of current research. Nevertheless, due to the rapid advancement in computing power, higher grid resolution mesoscale NWP models will be

able to adopt bulk microphysics parameterization schemes directly in the foreseeable future and reasonably resolve the mesoscale convective clouds and precipitating systems. However, many operational and research NWP models are initialized by global models which still rely heavily on CP schemes to represent the cumulus clouds, especially in areas with sparse data. Thus, some basic understanding of CP schemes is required for mesoscale modelers to interpret modeling results because some simulated features might have been implicitly inherited from the use of CP schemes in the large-scale or global models.

Existing parameterizations of cumulus convection for large-scale models may be divided into two groups: (a) equilibrium between mass or moisture supply and consumption is taken as the guiding principle, and (b) a balance between energy supply and consumption is postulated (Raymond 1994). The first group includes Kuo (1965) schemes and the second group includes convective adjustment schemes (e.g., Manabe et al. 1965, Kurihara 1973; Betts and Miller 1993) and Arakawa-Schubert (Arakawa and Schubert 1974) scheme. Schemes developed for mesoscale models include the Kain-Fritsch (Kain 2004) and Grell (1993) schemes, among others. In the following, we will briefly describe the convective adjustment scheme and Kuo schemes, which are presented in order to help understand the basic concepts of cumulus parameterization, as well as some of the schemes developed for mesoscale models.

a. Convective adjustment schemes

Convective adjustment refers to the concept that an unstable lapse rate cannot persist in the atmosphere and tends to be removed by either dry or moist convection. Thus, it is

plausible to assume that it will do so by adjusting the vertical stratification toward a state that is approximately neutral for moist convection. If the time scale of convection is much smaller than that of circulations resolved in a numerical model, then an instantaneous adjustment to a neutral state can be applied as a first approximation. In addition, convective processes associated with cloud ensembles in nature are complex and are normally represented through a subgrid scale in mesoscale NWP models. Any attempts to simulate their integrated effects by defining the actual properties of subgrid-scale clouds require the use of many arbitrary parameters, whose values are poorly known in nature and that require enormous amounts of computing power. Based on these arguments, convective adjustment is a conceptually simple and straightforward approach in which the explicit convective processes do not need to be simulated. Figure 14.8 shows four vertical profiles of virtual potential temperature θ_v observed from four tropical regions of strong convection. The four soundings in Fig. 14.8 resemble moist adiabats to some degree.

The convective adjustment scheme may be further divided into the following groups: (a) *hard convective adjustment* schemes, (b) *soft convective adjustment* schemes, and (c) *time-dependent convective adjustment* schemes. In the hard convective adjustment scheme, the convective adjustment is involved only within layers that are saturated and convectively unstable (Manabe et al. 1965). In the hard convective adjustment scheme, an initial large-scale sounding in which $\partial\theta_e/\partial p > 0$, is adjusted so that θ_e, or equivalently, moist static energy h $(= c_p T_s + gz_s + Lq_s)$, is set to be constant with height (Krishnamurti et al. 1980). In order to overcome the problem of rainfall rate overprediction, the so-called *soft convective adjustment scheme* is proposed. In the soft

convective adjustment scheme, saturation is assumed to occur only over a small fraction of the large-scale grid area, with the air between the clouds remaining unchanged. For example, the saturation is defined by 80% relative humidity by Miyakoda et al. (1969). In this way, convective adjustment is allowed to occur prior to grid-scale saturation and requires an unsaturated final state. The use of instantaneous convective adjustment, no matter hard or soft adjustment, suffers some drawbacks (Frank and Molinari 1993). For example, due to the lack of explicit simulation of the cloud properties, the convective effects on momentum field associated with production and transport of hydrometeors cannot be described by simply adjusting the lapse-rate. In addition, regions of potential instability are removed too quickly at mesoscale.

The above hard and soft convective adjustment schemes are improved by computing the depth of the adjusted layer using parcel concepts rather than localized grid-scale instability. This type of convective adjustment schemes are often referred to as *time-dependent convective adjustment schemes*, and may be accomplished in a number of ways. For example, the the adjustment time is specified explicitly in the *Betts-Miller* (1986) *CP scheme*. The Betts-Miller CP scheme is found to be quite robust for a wide variety of applications, and can be adapted for the mesoscale by adjustment of several parameters. The drawbacks of Betts-Miller CP scheme include the fact that the closure adopted appears to be less appropriate in cases of explosive deep convection and does not directly generate meso-β scale highs and lows (Seaman 1999).

b. Kuo schemes

Based on observations, Kuo (1965) proposed that cumulus convection occurs in deep layers of conditionally unstable stratification over areas of mean low-level convergence. The cloud base is taken to be the lifting condensation level of the surface air, the vertical profiles of temperature distribution T_s and mixing ratio q_s follow a moist adiabat, and the cloud top is located at the level of neutral buoyancy (LNB). In addition, the cumulus clouds are assumed to dissolve immediately by mixing with the environmental air, imparting to it heat and moisture. The moisture cycle in an air column which contains convection in Kuo schemes is illustrated in Fig. 14.9. In the following, we will present the basic concept of the Kuo scheme.

The conservation equation of water vapor, (14.3.4), may be written in pressure coordinates, ignoring the detailed microphysical processes, and retaining only the vertical eddy flux of water vapor,

$$\frac{\partial q_v}{\partial t} + \nabla \cdot (q_v \boldsymbol{V}) + \frac{\partial (q_v \omega)}{\partial p} = -(c - e) - \frac{\partial \overline{q_v' \omega'}}{\partial p}, \tag{14.3.11}$$

where c is the condensation rate per unit mass of air, e the evaporation rate, and ω the vertical motion in the pressure coordinates. Integrating (14.3.11) vertically from the surface (p_s) to the top of the atmosphere ($p = 0$) leads to

$$M_v + E = \frac{1}{g} \int_0^{p_s} (c - e) dp + S_{qv}, \tag{14.3.12}$$

where M_v is the vertically integrated horizontal *moisture convergence*,

$$M_v = -\frac{1}{g} \int_0^{p_s} \nabla \cdot (q_v \boldsymbol{V}) dp. \tag{14.3.13}$$

In the above equation, E and S_{qv} are the surface evaporation rate and the storage rate of water vapor, respectively,

$$E = -\frac{1}{g}\left[\overline{q_v'\omega'}\right]_s, \tag{14.3.14}$$

$$S_{qv} = \frac{1}{g}\int_0^{p_s}\frac{\partial q_v}{\partial t}dp\,. \tag{14.3.15}$$

The relationship between the integrated net condensation and the precipitation rate can be obtained by considering the conservation of cloud water q_c,

$$\frac{\partial q_c}{\partial t} + \nabla\cdot(q_c\boldsymbol{V}) + \frac{\partial(q_c\omega)}{\partial p} = (c-e) - P_{CR} - \frac{\partial\overline{q_c'\omega'}}{\partial p}, \tag{14.3.16}$$

where P_{CR} is the conversion rate of cloud water to precipitation. Integrating (14.3.16) with respect to pressure gives

$$\frac{1}{g}\int_0^{p_s}(c-e)dp = P + S_{ql} - M_l\,, \tag{14.3.17}$$

where P is the precipitation rate, S_{ql} is the storage rate of liquid water and M_l is the vertically integrated horizontal convergence of cloud water. Substituting (14.3.17) into (14.3.12) yields

$$M_v + M_l + E = P + S_{qv} + S_{ql}\,. \tag{14.3.18}$$

Equation (14.3.18) describes the moisture budget in which the sources of water vapor and cloud water into a unit air column are balanced by the precipitation plus storage of water vapor and liquid water. If one assumes that the convergence of cloud water (M_l) is much smaller compared to ($M_v + E$), and the storage term of the water vapor is negligible, then

the net rainfall rate plus the storage rate of cloud water is equal to the sum of large-scale moisture convergence and evaporation,

$$P + S_{ql} = M_v + E \,. \tag{14.3.19}$$

The above approximation is reasonably good over a relatively longer time scale, although substantial changes in the storage terms cannot be ignored at a short time scale. If the surface evaporation rate (E) is parameterized by the conventional bulk formula, then we have

$$M_t \equiv M_v + E = -\frac{1}{g}\int_0^{p_s} \nabla \cdot (q_v \boldsymbol{V}) dp + \rho_s C_d V_s (q_{ss} - q_s) \,, \tag{14.3.20}$$

where M_t is the *moisture accession,* ρ_s is the surface air density, C_d the drag coefficient, V_s is the near surface wind speed, q_{ss} the saturation mixing ratio at the sea surface temperature and pressure, and q_s the near surface saturation mixing ratio. The cumulus convection in the Kuo schemes is driven primarily by the moisture convergence.

The large-scale equation of thermodynamics for potential temperature in pressure coordinates may be written as

$$\frac{\partial \theta}{\partial t} + \nabla \cdot (\boldsymbol{V}\theta) + \frac{\partial (\omega\theta)}{\partial p} = \frac{1}{\pi}\left(L(c-e) - \frac{\partial \overline{\omega'\theta'}}{\partial p} + Q_r \right), \tag{14.3.21}$$

where L is the latent heat of condensation for water vapor, Q_r the radiative heating rate, and π the *Exner function* as defined in (13.5.16). The horizontal eddy flux of sensible heat is ignored in the above equation. We may define the net *cumulus heating* as

$$Q_c = L(c-e) - \frac{\partial \overline{\omega'\theta'}}{\partial p} \,, \tag{14.3.22}$$

which is part of the right hand side of (14.3.21). Based on Kuo's (1965) original approach, taking the vertical integration of (14.3.22), neglecting the sensible heat flux term, and using (14.3.19) and (14.3.20) lead to

$$\int_0^{p_s} Q_c dp = gLM_t \,. \tag{14.3.23}$$

The vertical structure of Q_c is assumed to be in the form of a relaxation toward a *moist adiabat* θ_{ma},

$$Q_c = \frac{\pi(\theta_{ma} - \theta)}{\tau}, \tag{14.3.24}$$

where τ is a relaxation time that is potentially a function of *x, y*, and *t*, but not of *p*. In regions where $M_t < 0$ and $\theta > \theta_{ma}$, Q_c is set to zero. From (14.3.24) and (14.3.23), the relaxation time may be estimated by the following equation,

$$\tau = \frac{1}{gLM_t}\int_0^{p_s} \pi(\theta_{ma} - \theta)dp \,. \tag{14.3.25}$$

Thus, the relaxation time is inversely proportional to the moisture accession.

The moisture convergence can be divided into bM_t, which increases the humidity of the air column, and $(1-b)M_t$, which is condensed and precipitate as rain (Kuo 1974). Normally, b is much less than 1 and should depend on the mean relative humidity of the air column

$$b = \left[\frac{1-RH}{1-RH_c}\right]^n \qquad RH \geq RH_c \tag{14.3.26}$$

$$= 1 \qquad\qquad RH < RH_c \,,$$

where RH_c is a critical value of relative humidity and *n* is a positive exponent of order 1 which may be empirically determined (Anthes 1977). The modified form of the Kuo scheme is also known as the *Anthes-Kuo scheme*. Kuo and Anthes (1984) found that the best agreement between observed and diagnosed rainfall rates is when *n* is between 2 and 3 and RH_c is between 0.25 and 0.50. An alternative method for estimating *b* is (Krishnamurti et al. 1980)

$$b = -\frac{1}{gM_t}\int_0^{p_s} \nabla \cdot (q_v \boldsymbol{V}) dp \,. \qquad (14.3.27)$$

In the *Arakawa-Schubert* (1974) *scheme*, a spectrum of cloud types is considered and the scheme is coupled with a model of the mixed layer. Further, the large-scale forcing function involves horizontal and vertical advection, radiation, and surface fluxes of heat and moisture (rather than only large-scale moisture convergence as in Kuo schemes).

c. Cumulus parameterization schemes for mesoscale models

One of the most challenging problems in mesoscale modeling is the parameterization of cumulus clouds. The conceptual basis for cumulus parameterization requires, in principle, the existence of a spectral gap between the scales being parameterized and those being resolved on the grid points. The spectral gap ensures that all eddies have a time scale much smaller than the grid-scale motions, so that their integrated influence can be incorporated into a single time step. For mesoscale models with grid resolution of 10 to 50 km and time intervals in the order of several minutes, mesoscale circulations appear to be resolved reasonably well, but the models are still not fine enough to resolve

cumulus convective clouds. In this situation, the convective scales and the resolvable scales are no longer as distinguishable as that assumed in the cumulus parameterization schemes adopted by large-scale models. Thus, the traditional cumulus parameterization schemes, such as Kuo schemes and the Arakawa-Schubert scheme, are not suitable for mesoscale models. The success of cumulus parameterization in numerical simulations of mature hurricanes using 10-20 km grid resolution is due to the fact that under strong rotation, the local *deformation radius* can shrink enough to produce a long-lasting, inertially stable disturbance. Thus, the time-scale separation requirement is met (Ooyama 1982).

Three approaches have been taken for the simulation of cumulus convection in mesoscale models (Molinari 1993): (1) the *traditional approach,* which utilizes cumulus parameterization, as those adopted by large-scale models, at convectively unstable grid points and explicit condensation at convectively stable grid points, (2) the *grid explicit approach,* which uses only explicit representations or bulk microphysics parameterization of microphysical processes regardless of stability, and (3) the *hybrid approach,* which parameterizes convectively unstable updrafts and downdrafts at convectively unstable grid points and also detrains a fraction of the parameterized cloud and precipitation particles to their respective grid-scale prediction equations. Due to the continuous advances in computer speed and memory, the grid explicit approach with microphysics parameterization schemes may become more applicable in resolving the cumulus parameterization problems in mesoscale models. In the following, we will briefly introduce some mesoscale cumulus parameterization schemes, such as the Kain-Fritsch scheme and the Grell scheme.

The *Kain-Fritsch* (*KF*) *scheme* (Kain and Fritsch 1993; Kain 2004) is a mass flux parameterization that uses the Lagrangian parcel method to estimate whether instability exists, whether any existing instability will become available for cloud development, and what the properties of any convective clouds might be. The scheme involves three parts: (1) the convective trigger function, (2) the mass flux formulation, and (3) the closure assumptions. The first part of the KF scheme is to identify potential source layers for convective clouds or *updraft source layers* (USLs). Beginning at the surface, vertically adjacent layers in the model are mixed until the depth of the mixed layer is at least 60 hPa. The combination of adjacent model layers composes the first potential USL which may be viewed as an "air parcel". The mean thermodynamics characteristics of this mixed layer are computed along with the temperature and height of this "air parcel" at its lifting condensation level (LCL). The potential of convective initiation is measured by $T_{LCL} - \overline{T}$ ($\overline{T}$ is the environmental temperature), which is typically negative indicating a negative buoyant air parcel. Observations suggest that convective initiation tends to be favored by background vertical motion (Fritsch and Chappell 1980). Thus, the parcel is assigned a temperature perturbation (δT_{vv}) linked to the magnitude of grid-resolved vertical motion, such as

$$\delta T_{vv} = k\left[w_g - c(z) \right]^{1/3}, \tag{14.3.28}$$

where k is a unit number with dimensions $\mathrm{K\,cm^{-1/3}s^{1/3}}$, w_g is an approximate running-mean grid-resolved vertical velocity ($\mathrm{cm\,s^{-1}}$) at the LCL, and $c(z)$ is a threshold vertical velocity given by

$$c(z) = \begin{cases} w_o(z_{LCL}/2000), & z_{LCL} \le 2000 \\ w_o, & z_{LCL} > 2000, \end{cases} \tag{14.3.29}$$

where $w_o = 2\,\mathrm{cm\,s^{-1}}$ and z_{LCL} is the height of LCL above the ground in m. Equation (14.3.29) will effectively eliminate most air parcels (mixed layer) as candidates for deep convection. If $T_{LCL} + \delta T_{vv} < \overline{T}$, then this mixed layer is excluded for deep convection, the base of the USL is moved up one model level, and the above test is repeated for a new potential USL. Otherwise, the mixed layer or air parcel is allowed to proceed as a candidate for deep convection. At this stage, the parcel is released at its LCL with its original temperature and moisture content and a vertical velocity derived from the perturbation temperature, such as

$$w_{po} = 1 + 1.1\left[\left(z_{LCL} - z_{USL}\right)\delta T_{vv}/\overline{T}\right]^{1/2}, \tag{14.3.30}$$

where z_{USL} is the height at the base of the USL. The above equation yields initial vertical velocity for the air parcel up to several meters per second. Above the LCL, parcel vertical velocity is estimated at each model level using the Lagrangian parcel method (e.g., Perkey and Kreitzberg 1993), including the effects of entrainment, detrainment, and water loading (Bechtold et al. 2001). If vertical velocity remains positive over a depth that exceeds a specified minimum cloud depth (typically 3-4 km), deep convection is activated using this USL. If deep convection is not activated, the base of the potential USL is moved up one model layer and the procedure is repeated until either the first suitable source layer is found or the sequential search has moved up above the lowest 300 hPa of the atmosphere. The set of criteria described here gives the *trigger function*. Note

that the cloud depth is determined by the updraft model described in the mass flux formation which, in turn, determines whether the parameterization is activated.

The second part of the KF scheme is the *mass flux formation*. In this scheme, convective updrafts are represented using a steady-state entraining-detraining plume model, where both θ_e and q_v are entrained and detrainted. Convective downdrafts are generated by evaporation of condensate that is produced within the updraft. A fraction of this total condensate is made available for evaporation within the downdraft, based on empirical formulas for precipitation efficiency as a function of vertical wind shear and cloud-base height. The downdraft is specified to start at the level of minimum θ_e^* in the cloud layer with a mixture of updraft and environmental air. It is moved downward in a Larangian sense, with a specified entrainment rate and a fixed relative humidity of 100% above cloud base and 90% below cloud base. When the downdraft is warmer than its environment, it is terminated and forced to detrain into the environment within and immediately above the termination level. The scheme also requires environmental mass fluxes to compensate for the upward and downward transports in updrafts and downdrafts so that there is no net convective mass flux at any level in the column. The third part of the KF scheme is the *closure assumptions*. The KF scheme rearranges mass in a column using the updraft, downdraft, and environmental mass fluxes until at least 90% of the CAPE is removed. CAPE is computed in the traditional way (see Ch. 7) and is removed by the combined effecs of lowering θ_e in the USL and warming the environment aloft. The *convective time scale*, or *relaxation period* (τ_c), is based on the advective time scale in the cloud layer, which has upper and lower limits given as $0.5h \le \tau_c \le 1h$. The scheme

feeds back convective tendencies of temperature, water vapor mixing ratio, and cloud water mixing ratio.

Another widely adopted cumulus parameterization scheme developed mainly for mesoscale models is the *Grell scheme* (Grell 1993). The key features of the Grell scheme are (Seaman 1999): (a) deep convective clouds are all of one size; (b) the Arakawa-Schubert (1974) cloud work function was adopted for its closure, but this was later changed to use a CAPE closure, similar to that in Kain-Fritsch scheme; (c) no lateral mixing (i.e. no entrainment or detrainment) except at the levels of origin or termination of updrafts and downdrafts, thus making mass flux constant with height; and (d) it is not necessary to assume that the fractional area coverage of updrafts and downdrafts in the grid column is small since there is no lateral mixing. The absence of lateral mixing allows the scheme to operate relatively easier at finer scales, although some degree of scale separation is still important. The Grell scheme has been modified, based on some features developed in the Kain-Fritsch scheme. The advantages of the Grell scheme are that it includes effects of downdrafts and is well adapted for grids as fine as 10 to 12 km.

14.4 Parameterizations of radiative transfer processes

14.4.1 Introduction

Solar radiation is a major driving force of the atmospheric motion. The magnitude of radiative warming/cooling depends on many factors, including temperature, clouds, aerosols, water vapor, carbon dioxide, and ozone. Radiation is also the primary force for the soil model in terms of the surface energy budget. Figure 14.10 illustrates the radiative transfer processes in the Earth-atmosphere system, which include shortwave and

longwave reflection, transmission, and absorption/emission. The radiative transfer processes are very complex and do not allow mesoscale modelers to make detailed adjustments due to limitations of computing time. Thus, similar to PBL and moist processes, the radiative transfer processes are parameterized in mesoscale models. The purpose of this section is to introduce the basic concepts of the parameterization of radiative transfer processes for mesoscale and numerical weather prediction models. Detailed discussions on detailed radiative transfer processes and parameterizations can be found in advanced textbooks (e.g., Liou 1992, 2002).

The objective of parameterizing the atmospheric radiative transfer processes in a numerical model is to provide a simple, accurate and fast calculation of the total radiative flux profile within the atmosphere. The fast, simple calculation includes (i) the total radiative flux at the surface to calculate the surface energy balance, and (ii) the vertical radiative flux divergence to calculate the radiative warming and cooling rates of an atmospheric volume. The parameterization commonly includes the combined effects of absorption/emission and scattering by the radiatively active trace-gases of H_2O, CO_2, and O_3, together with cloud and haze particles.

Different levels of approximation have also been adopted, which depend on the desired accuracy for representing the type of interactions between radiation and dynamics. The factors often considered in selecting the parameterization include (Stephens 1984): (a) radiation may simultaneously affect the dynamics in several different ways, and the accuracy required of the radiation computations depends on which process is important to the given dynamical problem. (b) The dynamics respond to the total heating fields, which include radiative, latent and sensible heating. The heating

components are not always independent of each other and, as a result, radiation may influence the dynamics in a complex nonlinear manner that is often difficult to assess *a priori*. (c) The radiative warming and cooling may vary considerably in response to variations in temperature, which are caused by various dynamical factors.

Atmospheric radiation covers a broad spectrum of electromagnetic waves. As depicted in Fig. 14.11, the sun radiates approximately at a blackbody temperature of 6000 K, which spans the entire spectrum of the electromagnetic waves, but the radiation outside the range 0.2 – 4.0 μm is negligible and referred to as the *shortwave radiation*. Likewise, the earth emits radiation at a blackbody temperature of about 250 K, which covers the entire spectrum of the the electromagnetic waves, but the radiation outside the range 4 – 200 μm is negligible, and is referred to as the *longwave radiation*. The *blackbody* emission of electromagnetic radiation for a particular wavelength λ and temperature T may be derived,

$$B_\nu(T) = \frac{2h\nu^3 c^2}{e^{hc\nu/KT} - 1}, \tag{14.4.1}$$

where B_ν is called the *Planck function*, h = 6.6262 x 10^{-34} J s is the *Planck constant*, K = 1.380 x 10^{-23} J K^{-1} is the *Boltzmann's constant*, ν is the wave number ($\nu = 1/\lambda$, λ is the wavelength), and c is the speed of light. The *total radiative flux, F,* emiting at a blackbody temperature T may be derived by integrating (14.4.1) over the entire spectral range and angles,

$$F = \sigma T^4, \tag{14.4.2}$$

where $\sigma = 5.67032 \text{x} 10^{-8}$ $\text{Wm}^{-2}\text{K}^{-4}$ is the *Stefan-Boltzmann constant*. The above equation is also known as the *Stefan-Boltzmann law*. However, the earth has an atmosphere which

contains molecules and particulates. Thus, it does not behave as a blackbody, which leads to

$$F = \varepsilon \sigma T^4, \tag{14.4.3}$$

where ε denotes the *emittance*. The emittance, ranging from 0 to 1, represents the ratio of the flux emitted by a graybody to that by a blackbody at the same temperature. Satisfying the condition of local thermal equilibrium, the emitance of a medium is equal to its absorptance, which is also known as *Kirchhoff's law of thermal radiation.*

The *net radiation heating* may be written as,

$$\frac{\partial \theta}{\partial t} = \frac{1}{\overline{\rho} c_p} \frac{dF_N}{dz}, \tag{14.4.4}$$

where $\partial\theta / \partial t$ has also been written as $\partial T / \partial t$ in the literature, F_N is the difference between downward ($F^{\downarrow}$) and upward ($F^{\uparrow}$) fluxes (in W m^{-2}) and dF_N / dz is the *vertical flux divergence*.

When electromagnetic radiation traverses a layer in the atmosphere, it can be transmitted, absorbed, or reflected. Based on the conservation of energy, it can be derived that

$$T_\nu + A_\nu + R_\nu = 1, \tag{14.4.5}$$

where T_ν, A_ν, and R_ν are *transmissivity*, *absorptivity*, and *reflectivity*, respectively. A pencil of radiation traversing a medium may be weakened by extinction (i.e., *scattering* + *absorption*) of the material, strengthened by emission of the material, or undergo multiple scattering from all other directions into the pencil (Liou 2002). Thus, the change of the *intensity of radiation* traversing a medium may be expressed by

$$\left(\frac{1}{k_\nu \rho_a}\right)\frac{dI_\nu}{ds} = -I_\nu + B_\nu(T) + J_\nu, \qquad (14.4.6)$$

where k_ν denotes the *extinction coefficient*; ρ_a, the *density* of media; I_ν, the *incident radiance*; $B_\nu(T)$, the *thermal emission;* and J_ν, the *source of radiation* from scattering into the line segment *ds* (Fig. 14.12). In the absence of emission and scattering, the *Beer-Bouger-Lambert law* can be derived, which states that the radiant intensity trasversing a homogeneous extinction medium decreases exponentially as $I_\nu = I_{\nu o}\exp(-k_\nu u)$, where u is the *path length*.

14.4.2 Longwave radiation

a. Clear atmosphere

In a clear air atmosphere, the scattering of longwave radiation (J_ν) may be neglected compared with the absorption and emission (Liou 2002). More specifically, the general problem of parameterizing the longwave radiative transfer in the clear air requires the suitable treatment of absorption and simultaneous emission by the ozone band (9.6 μm), the rotation and vibration bands of water vapor, the continuum absorption in the atmospheric window (between 8 μm and 14 μm) and the absorption by the carbon dioxide band (15 μm), which overlaps a portion of the rotation band (Stephens 1984).

By considering a monochromatic radiation of wavelength λ entering at angle θ (Fig. 14.12) from the vertical direction across a plane sheet of material of distance ds and vertical distance *dz*, (14.4.6) becomes

$$\left(\frac{\mu}{k_\nu \rho_a}\right)\frac{dI_\nu}{dz} = -\mu\frac{dI_\nu}{d\tau} = -I_\nu + B_\nu(T)\,, \tag{14.4.7}$$

where

$$\tau = \int_z^\infty k_\nu \rho_a \, dz\,, \tag{14.4.8}$$

is the *normal optical thickness* (or simply *optical thickness*), $\mu\,(=\cos\theta)$ is the cosine of the zenith angle, and dI_ν/dz is the intensity change in the vertical direction. In order to solve the first-order differential equation, (14.4.7), for both upward and downward components for an atmosphere with a total optical thickness of τ_*, two boundary conditions are required at the surface and the top of the atmosphere. For the atmospheric heating rate $(\partial\theta/\partial t)$ calculations, the required quantities are the upward and downward radiative fluxes ($F^\uparrow$ and $F^\downarrow$), as expressed in (14.4.4). The radiative fluxes at a particular wave frequency ($F_\nu^\uparrow$ and $F_\nu^\downarrow$) can be calculated by taking the integration of the intensities of radiation ($I_\nu^\uparrow$ and $I_\nu^\downarrow$) with respect to μ. The upward and downward radiative fluxes ($F^\uparrow$ and $F^\downarrow$) can then be obtained by taking integrations of $F_\nu^\uparrow$ and $F_\nu^\downarrow$, respectively, from $\mu = 0$ to 1.

Based on above equations with the absence of scattering, the *longwave radiative flux* may be derived to be

$$F^\uparrow(z) = \int_0^\infty \pi B_\nu(z=0)\tau_\nu^f(z, z=0)d\nu + \int_0^\infty\int_0^z \pi B_\nu(z')\frac{\partial}{\partial z'}\tau_\nu^f(z,z')dz'd\nu\,, \tag{14.4.9}$$

$$F^\downarrow(z) = \int_0^\infty\int_z^\infty \pi B_\nu(z')\frac{\partial}{\partial z'}\tau_\nu^f(z,z')dz'd\nu\,, \tag{14.4.10}$$

where $F^{\uparrow}(z)$ and $F^{\downarrow}(z)$ are the *upward and downward longwave radiative flux* through level *z*, B_ν is the *Planck function* in terms of wave frequency and the *diffusion transmission function*, τ_ν^f, is defined by the hemispheric integral

$$\tau_\nu^f(z,z') = 2\int_0^1 \tau_\nu(z,z',\mu)\mu d\mu, \tag{14.4.11}$$

and

$$\tau_\nu(z,z',\mu) = \exp\left[-\frac{1}{\mu}\int_{u(z)}^{u(z')} k_\nu(p,T)du\right], \tag{14.4.12}$$

where $k_\nu(p,T)$ is the *absorption coefficient* and u is the *absorption* or *optical path* from z to z'. Note that *u* should be the mass if the unit *k* is in the form fraction/mass. The transmission function τ_ν^f is often referred to as the *slab transmission function,* as described in (14.4.11). The above equations may be combined and differentiated directly to obtain an equation of flux divergence, and thus of radiative cooling. However, for aplications in a mesoscale or general circulation model, these fluxes may be calculated numerically at each model level, followed by the evaluation of the flux divergence for the layer between two levels. The evaluation of flux divergence between two levels automatically supplies fluxes at those levels where a radiation budget is required, such as at the earth surface, the tropopause or at the model top.

There are three integrals involved in the calculations of *longwave flux*, which include (14.4.11), the inner integrations of (14.4.9) and (14.4.10) over all atmospheric layers (dz'), and the outer integrations of (14.4.9) and (14.4.10) over all spectral intervals ($d\nu$). The integration of (14.4.11) may be approximated by (Stephens 1984)

$$\tau_\nu^f(z,z') \propto \tau_\nu(z,z',1/\beta), \qquad (14.4.13)$$

where $\beta = 1.66$ and is known as the *diffusivity factor*. Basically, this approximation means the diffuse transmission can be approximated by intensity transmission with an angle of $\cos^{-1}(1/\beta)$. The integration of (14.4.9) and (14.4.10) over dz' may be approximated by a finite difference or other numerical method in the vertical direction. Thus, the objective of *parameterization of longwave radiation in a clear sky* is to find suitable approximations of integration over z' and ν in (14.4.9) and (14.4.10) and the integration over μ in (14.4.1).

The difficulties in the *integration over optical path* in (14.4.12) are due to the fact that the absorption coefficient k_ν is a function of both pressure and temperature, and most absorption data are collected in the laboratory at constant pressure and temperature, which are not necessarily applicable to the real atmosphere. Two commonly adopted approximations are (a) the *one-parameter scaling approximation* (Goody 1964a; Chou and Arking 1980) and (b) the *two-parameter approximation* (Goody 1964b). The one-parameter scaling approximation method is able to provide a reasonable approximation to the problem of absorption along nonhomogeneous paths. Difficulties arise when one attempts to isolate the errors in the infrared cooling rate, which are likely to be larger than those of the two-parameter approximation method, especially in the upper atmosphere.

The problem inherent in simplifying the *integration over frequency* in (14.4.9) and (14.4.10) is more complicated than the rather simple and obvious task of averaging k_ν over some broad interval $\Delta\nu$. The finest frequency scale of absorption, that of an

individual line, is described by a simple analytical function given by the Lorenz line absorption profile for an altitude below about 40 km (e.g., Liou 2002). Unfortunately a single absorbing line cannot be considered in isolation from neighboring lines, and it is not meaningful to average k_ν over a group of lines in a simple linear manner because k_ν is the sum of the contributions to the absorption coefficient at a given frequency from all lines. The properties of the single absorption line leads to the concept of a *band model*, which enables the averaging of the absorption properties for bands of lines, such as line strength, separation and position that are specified by well-defined statistical relationships.

An alternative approach is the k (absorption coefficient)-*distribution method*, which makes use of the fact that for a homogeneous atmosphere, transmission within a relatively wide spectral interval depends only on the fraction of the interval that is associated with a particular value of k. The k-distribution method has been demonstrated to be faster and more accurate than the band model (e.g., Arking and Grossman 1972; Chou and Arking 1980). In addition, treating molecular absorption and the scattering by cloud droplets in a self-consistent fashion in the k distribution method is straightforward. Like other band-model methods, the k-distribution method was developed for homogeneous atmospheres. For nonhomogeneous atmospheres, the one-parameter scaling approximation is often adopted in the integration of (14.4.12) over optical path, which may lead to large deviation from results computed from line-by-line (LBL) methods (e.g. Rothman et al. 1987). In order to overcome this problem, the correlated k-distribution method has been proposed (e.g. Fu and Liou 1992; Mlawer et al. 1997).

More details in recent developments of the *k* distribution method can be found in Kratz et al. (1998) and Chou et al. (1999).

b. Cloudy atmosphere

The optical path length for infrared or longwave radiation is strongly influenced by the presence of clouds. Mesoscale models have often treated clouds as blackbodies in the longwave portion of the spectrum, where no infrared radiation is transmitted through the cloud. Although more advanced schemes have been developed, the parameterization for longwave radiation within a water cloud can be understood through the following scheme (Stephens 1978a):

$$F^{\uparrow}(z) = F^{\uparrow}(z_b)[1-\varepsilon_b] + \varepsilon_b \sigma T_c^4, \text{ and} \qquad (14.4.14a)$$

$$F^{\downarrow}(z) = F^{\downarrow}(z_t)[1-\varepsilon_t] + \varepsilon_t \sigma T_c^4, \qquad (14.4.14b)$$

where $F^{\uparrow}(z_b)$ and $F^{\downarrow}(z_t)$ are the clear-air radiative flux at the cloud base (z_b) and cloud top (z_t), respectively, and T_c is the cloud temperature. The *cloud effective emissivity* $\varepsilon_b(z_b, z)$ and $\varepsilon_t(z_t, z)$ can be obtained by solving (14.4.14) using a detailed radiational model with eight cloud types in a U.S. standard atmosphere to obtain $F^{\uparrow}(z_b)$ and $F^{\downarrow}(z_t)$ (Stephens 1978b). Liou and Ou (1981) also proposed a parametarization of longwave radiative transfer in the presence of a semitransparent cloud layer. They used a model with five broadband emissivity values to represent the five major absorption regions in the infrared spectrum. For application to a mesoscale model, (14.4.14) can be used when a grid volume is saturated with clouds (Pielke 2002).

14.4.3 Shortwave radiation

a. Clear atmosphere

The distribution of electromagnetic radiation emitted by the sun is approximately as blackbody radiation for a temperature of about 6000 K. The spectral distribution of solar or shortwave radiation received at sea level through a cloud free and haze free atmosphere is shown in Fig. 14.11. As can be seen in the figure, the primary absorptions in a clear atmosphere are by: (1) the ozone in the visible ($0.5\mu m \le \lambda \le 0.7\mu m$) and ultraviolet ($\lambda \le 0.3\mu m$) spectra; and (2) the water vapor in the near infrared (IR) spectrum ($0.7\mu m \le \lambda \le 4.0\mu m$). Thus, $\lambda = 0.7\mu m$ is a natural division of these two absorbers. The major absorption bands are shaded in Fig. 14.11. The absorptions by O_2 and CO_2 are substantially less than that of ozone and water vapor and their contributions can be ignored. The *solar irradiance* is composed by *direct irradiance* and *diffuse irradiance*. The diffuse irradiance is the irradiance observed at a point from directions other than the line of propagation, while the direct irradiance is the irradiance observed at a point without being absorbed or scattered from its line of propagation.

In the absence of scattering, the downward shortwave irradiance through level z for a collimated beam of solar irradiance may be derived,

$$F_{sw}^{\downarrow}(z,\mu_o) = \mu_o \int_0^{\infty} S_{o\nu}\tau_{o\nu}(z,\mu_o)d\nu\,, \qquad (14.4.15)$$

where $F_{sw}^{\downarrow}(z,\mu_o)$ is the downward irradiance through level z for a collimated beam of solar irradiance ($S_{o\nu}$) at the top of the atmosphere, inclined at a zenith angle θ_o (or $\mu_o = \cos\theta_o$). The monochromatic transmittance function can be calculated from

$$\tau_{o\nu}(z,\mu_o) = \exp\left(-\frac{1}{\mu_o}\int_z^\infty k_\nu du\right). \qquad (14.4.16)$$

Calculation of the downward shortwave radiation (14.4.15) is less complicated than its longwave counterpart, (14.4.10) since it is not necessary to consider the complications imparted by simultaneous absorption and emission from layer to layer (Stephens 1984). The only variable that remains to be defined is the mean transmission function

$$\tau_{o\nu}(z,\mu_o) = \frac{1}{\Delta\nu}\int_{\Delta\nu} \exp\left(-m_r(\mu_o)\int_z^\infty k_\nu du\right) d\nu\,, \qquad (14.4.17)$$

where $1/\mu_o$ is replaced by the *mass factor* $m_r(\mu_o)$ in (14.4.16), which is identical to $1/\mu_o$ except for larger solar zenith angle. Empirical formulae for $m_r(\mu_o)$ have been proposed. For example, Rodgers (1967) proposed that $m_r = 35\mu_o(1224\mu_o^2+1)^{-1/2}$ for ozone, otherwise $m_r = 1/\mu_o$. Thus, the *clear-sky downward solar flux*, (14.4.15), transmitted to level *z* along θ_o can be approximated by

$$F_{sw}^{\downarrow} = \mu_o \sum_{i=1}^{N} S_{oi}\tau_{\bar{\nu}i}(u)\,. \qquad (14.4.18)$$

The approach given in (14.4.18) is called the *discrete band approach*. The subscript *i* denotes the *i*th spectral interval. The upward solar flux received at level *z* by reflection from the earth surface may be calculated in a similar fashion,

$$F_{sw}^{\uparrow} = \mu_o \sum_{i=1}^{N} \alpha_{gi} S_{oi}\tau_{\bar{\nu}i}(u^*)\,, \qquad (14.4.19)$$

where α_{gi} is the *surface albedo* for the *i*th spectral interval. The *path length* u^* is the effective total absorber amount traversed by the diffusely reflected radiation. Some useful empirical formulae have been proposed for estimating u^* (e.g., Lacis and Hansen 1974).

Note that the mean transmittance is commonly defined as a convolution of the transmission function and the extraterrestrial flux $S_{o\nu}$. The mean transmittance function over the entire solar spectrum may be calculated by

$$\overline{\tau}_o(z,\mu_o) = \frac{1}{S_o}\int_0^\infty S_{o\nu}\tau_{o\nu}(z,\mu_o)d\nu\,, \tag{14.4.20}$$

in which case the downward solar flux at level z is defined by

$$S^-(z) = \mu_o S_o \overline{\tau}_o(z,\mu_o)\,. \tag{14.4.21}$$

Using (14.4.20) and (14.4.21) to calculate the downward solar flux is also called the *broadband approach*, and has been used for calculating solar fluxes within the atmosphere. However, the broadband approach has hardly been used recently because Rayleigh scattering is important in clear atmospheres and is included in nearly all models. Furthermore, different values of land surface reflectivity are used for different spectral bands in most of the current models, which requires the division of the solar spectrum into multiple bands.

In the lower atmosphere, the absorption of solar radiation by water vapor is the major source of solar heating. As mentioned earlier, the absorption of water vapor is concentrated in the near IR spectrum, $0.7\ \mu\text{m} \le \lambda \le 4.0\ \mu\text{m}$ (see Fig. 14.11). In order to resolve the water vapor absorption and to apply Beer's law, the spectrum has to be divided into about a half million intervals (Chou 1992). The difficulty in parameterizing the water vapor absorption is due to the fact that: (1) the absorption fluctuates strongly within very narrow spectral intervals, (2) the absorption is complicated by the pressure and temperature dependencies, and (3) the absorption spectrum of water vapor overlaps with that of liquid water absorption. Ideally, line-by-line methods (e.g. Rothman et al.

1987) are required for achieving a high degree of accuracy in solar flux calculation, although it is computationally very expensive. In order to reduce the computational burden, there are numerous methods being proposed for parameterizing the absorption functions of the water vapor, such as the simple parameterization of the broadband absorption functions (e.g., Chou 1986; Lacis and Hansen 1974). In order to improve the accuracy, the *k-distribution method* has been proposed and applied in studies such as Chou (1986), where it was found that the solar near-infrared fluxes could be accurately computed with a maximum of nine values of k, instead of half a million spectral intervals, in each of the three near IR bands.

In the ultraviolet (UV) and visible spectrum ($\lambda < 0.7\,\mu\text{m}$), the primary absorption of the solar radiation is due to ozone. The absorption spectrum of ozone is continuous in nature and requires less spectral intervals than the near-infrared spectrum for accurate calculations of the solar radiation. However, it is desirable to reduce the number of spectral intervals due to the relatively wide range of the absorption spectrum of ozone. Molecular scattering is significant at the absorption spectrum of ozone, but fortunately, they do not overlap much in high altitudes. The ozone absorption occurs in the higher atmosphere, while the molecular scattering occurs in the lower atmosphere. Again, many simple parameterizations of the broadband absorption functions have been proposed. Chou (1986) divided the spectrum between 0.175 μm and 0.7 μm into 8 intervals and used a single mean value of k for each interval. When there are more than one absorber and scatterer in an atmospheric layer, the effective optical parameters are required for flux computation (Tsay et al. 1989).

b. Cloudy atmosphere

The interactions of clouds with solar radiation are extremely complicated since the attenuation of solar radiation includes scattering and absorption by wide spectrum of cloud droplets and ice crystals. Thus, in addition to making proper representation of solar radiative processes, one also has to represent or parameterize microphysical processes properly. In the absence of emission, the basic equation governing the *intensity of solar radiation* appropriate to a cloud medium may be written in the form

$$\mu \frac{dI(\tau,\mu)}{d\tau} = -I(\tau,\mu) + \frac{\widetilde{\omega}_o}{4\pi}\int_{-1}^{1} \overline{p}(\tau,\mu,\mu')I(\tau,\mu')d\mu' + \frac{S_o}{4\pi}\overline{p}(\tau,\mu,\mu_o)e^{-\tau/\mu_o}, \tag{14.4.22}$$

where τ is the optical thickness, $\widetilde{\omega}_o$ is the *single-scattering albedo*, $\overline{p}$ is the *scattering phase function*, $I(\tau,\mu)$ is the radiance along the angle μ $(=\cos\theta)$ and S_o is the solar flux associated with a collimated beam incident on the cloud top. The optical parameters are functions of frequency, but for simplicity they are not shown in the equation. The difference between (14.4.22) and the flux equations of long-wave radiation, such as (14.4.6), is as follows (Stephens 1984): First, the optical thickness now includes the contributions from scattering (τ_s) and absorption (τ_a) by cloud droplets, and by the intervening gas (τ_g),

$$\tau = \tau_s + \tau_a + \tau_g . \tag{14.4.23}$$

Second, the single-scattering albedo is included in (14.4.22), which is a ratio of the scattering optical thickness to the total optical thickness, i.e.,

$$\widetilde{\omega}_o = \tau_s / \tau . \tag{14.4.24}$$

Thus, $\widetilde{\omega}_o = 1$ for a nonabsorbing cloud and $\widetilde{\omega}_o = 0$ if the scattering is negligible. Third, the *intensity of solar radiation* includes the scattering phase function $\overline{p}(\tau, \mu, \mu')$, which characterizes the angular distribution of the scattered radiation field. For spherical cloud droplets, this function exhibits an intense peak in the forward direction and produces rainbow and glory effects in the backward direction (Liou 2002). The most commonly adopted formula for the scattering phase function is (Henyey and Greenstein 1941),

$$\overline{p}(\tau, \mu, \mu') = \frac{1 - g^2}{1 + g^2 - 2g\mu\mu'}, \tag{14.4.25}$$

where g is the *asymmetry factor*, which is defined as

$$g = \frac{1}{2}\int_{-1}^{1} \overline{p}(\tau, \mu, \mu')\mu d\mu . \tag{14.4.26}$$

Note that g varies from –1 for complete backscattering, to 0 for isotropic scattering, and to 1 for forward scattering.

In order to calculate the intensity of solar radiation with the presence of clouds, one needs to solve (14.4.22), which can be viewed in terms of three key properties: layer optical thickness (τ), layer single-scattering albedo ($\widetilde{\omega}_o$), and asymmetric parameter (g). Due to the above complications, it is necessary to make some approximations for solving (14.4.22). After some manipulation, (14.4.22) may be approximated by two simultaneous differential equations and lead to the *Eddington approximation*, *two-stream approximation*, or to four first-order differential equations and lead to the *four-stream approximation* (see reviews in Liou 2002 and Stephens et al. 2001). Fu et al. (1997) have proposed a hybrid two-stream and four-stream method, which has been adopted by some mesoscale models.

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Problems

14.1 Derive the mean equations of (14.1.3)-(14.1.7) by applying the Reynolds averaging.

14.2 Derive (14.2.16).

14.3 (a) Derive (14.2.17). (Hint: you need to integrate the reduced time-independent equation of (14.1.3)-(14.1.4) from surface at $z = 0$ to the top of the mixed layer, $z = h$) (b) Estimate $\overline{V}$ for a mixed layer of $\overline{u}_g = 10\ \text{ms}^{-1}$, $C_d = 1.5 \text{x} 10^{-3}$, $f = 10^{-4}\ \text{s}^{-1}$, and $h = 1$ km.

14.4 Obtain the solution (14.2.26) from (14.2.25) and the boundary conditions described in the text.

14.5 Derive (14.2.30).

14.6 Derive (14.3.18) by neglecting the sensible heat flux term in (14.3.16).

Table captions

Table 14.1: Key to Fig. 14.6. (Adapted after Lin, Farley, and Orville 1983)

Figure captions

Fig. 14.1: Schematic illustration of subgrid scale values of vertical velocity w' ($\mathrm{cm\,s^{-1}}$), potential temperature θ' (K), and the subgrid scale covariance $w'\theta'$ (K cm $\mathrm{s^{-1}}$). In this example, the grid averaged value of vertical motion is required to be approximately 0 (i.e. $\overline{w} = 0$), and $\overline{\theta} = 299.5\mathrm{K}$ is used. Both grid value averages are assumed to be constant over Δx. The grid-averaged subgrid-scale correlation $\overline{w'\theta'}$ is equal to 6.9 cm K $\mathrm{s^{-1}}$. (Adapted after Pielke 2002)

Fig. 14.2: Typical convective boundary layer profiles of (a) mean virtual potential temperature, (b) specific humidity, (c) wind speed ($\overline{V}$ and $\overline{V}_g$ denote mean wind speed and geostrophic wind speed respectively), (d) vertical heat flux, (e) vertical moisture flux, and (f) vertical momentum flux. (Adapted after Driedonks and Tennekes 1984)

Fig. 14.3: Typical stable boundary layer (SBL) profiles of (a) mean temperature, (b) potential temperature, (c) wind speed, and (d) specific humidity. (Adapted after Stull 1988)

Fig. 14.4: A sketch of the wind vectors of the Ekman spiral (14.2.26). The arrows show the wind vectors at non-dimensional height $\gamma z = \pi/6, \pi/3, \pi/2, 2\pi/3$, where γ is defined in (14.2.26). (Adapted after Batchelor 1967)

Fig. 14.5: Comparison of predictions of the virtual potential temperature profile using (a) TKE closure scheme (Adapted after Sun and Chang 1986) and (b) third-order closure scheme (Adapted after André et al. 1978) for Day 33 of the Wangara experiment against (c) observational data (Adapted after André et al. 1978). The local times are denoted by the numbers adjacent to the curves.

Fig. 14.6: A sketch of cloud microphysical processes in a bulk microphysics parameterization (LFO) scheme including ice phase. Meanings of the production terms (i.e., P terms) can be found in Table 14.1. (Adapted after Lin, Farley, and Orville 1983; Orville and Kopp 1977)

Fig. 14.7: Simulations of tropical cyclones using the LFO scheme (Fig. 14.6). Radius-height distributions of (a) vertical velocity (contours are 0, $\pm$ 1, 2.5, and 4 ms^{-1}; areas greater than 4 ms^{-1} are dark-shaded, and areas of downward motion are light-shaded.), and (b) tangential wind velocity (contours: 0, $\pm$ 5, 10, 15, 20 and 25 ms^{-1}; dark-shaded for higher than 25 ms^{-1}) for the warm-rain numerical simulation at 22 h. (c) and (d) are the same as (a) and (b), respectively, but for the ice-phase simulation at 36 h. The dashed lines in (c) and (d) denote the melting level. (Adapted after Lord et al. 1984)

Fig. 14.8: Vertical profiles of virtual potential temperature (θ_v) for four tropical soundings. AMEX 5 is a composite sounding from the late decay stages of four cloud clusters (dashed), while PRC mean (thin line) is a six-week mean sounding from a ship during the Australian Monsoon Experiment (AMEX). GATE ST 6 (thick line) is a composite for the decay stages of eight GATE cloud clusters, and GATE ph

3 (dotted) is the phase III mean for the GATE array. (Adapted after Frank and Molinari 1993)

Fig. 14.9: A schematic for moisture cycle in a column which contains convection in Kuo schemes. See text for details. (Adapted from Anthes 1977)

Fig. 14.10: Illustration of global energy balance through radiative transfer processes in the Earth-atmosphere system, based on data obtained from NASA Earth Observing System and various published model and empirical estimates. (Courtesy of Dr. S.-C. Tsay)

Fig. 14.11: Spectral energy curves of solar (shortwave) and terrestrial (longwave) radiation observed, as well as modeled, under cloud-free conditions at sea level and at the top of atmosphere (after Tsay et al. 1989). The dark (solar) and dip (terrestrial) areas depict radiatively active gaseous absorption and emission bands while the shaded area represents Rayleigh and aerosol scattering effects. (Adapted after Tsay et al. 1989)

Fig. 14.12: A schematic illustration of radiant intensity attenuated by scattering and/or absorbing media between ds (= $\sec\theta\, dz$).