

Lecture 10 Adiabatic Processes

(Sec.3.5 of Hess) [Classical equation editor: $dq = 0$]

- **Definition:** If a thermodynamic system changes its physical state (i.e. p , α , and/or T) without any heat being either added to or extracted from the system, the changing process is said to be **adiabatic**.

Mathematically, adiabatic processes can be simply defined by

$$dq = 0$$

Notice that **the first law of thermodynamics can be written in various forms**, such as

$$dq = du + dw,$$

$$dq = c_v dT + p d\alpha,$$

$$dq = dh - \alpha dp,$$

$$dq = c_p dT - \alpha dp.$$

Therefore, the first law of thermodynamics for an adiabatic process reduces to the following,

$$c_v dT + p d\alpha = 0 \tag{3.5.1}$$

or

$$c_p dT - \alpha dp = 0. \quad (3.5.2)$$

Eq. (3.5.2) constrains the relations among the three state variables p , α , and T for adiabatic processes. We also know that these three state variables are constrained by the equation of state for ideal gases

$$p\alpha = RT,$$

or

$$\alpha = \frac{RT}{p}. \quad (3.5.3)$$

Substituting (3.5.3) into (3.5.2), we get

$$c_p dT - \frac{RTdp}{p} = 0. \quad (3.5.4)$$

Dividing Eq. (3.5.4) by RT leads to,

$$\frac{c_p}{RT} dT - \frac{dp}{p} = 0,$$

or

$$(c_p / R) d \ln T - d \ln p = 0, \quad (3.5.5)$$

Integrate (3.5.5) from the initial state (T_o, p_o) to any state (T, p) , we have

$$\frac{c_p}{R} \int_{T_o}^T d \ln T - \int_{p_o}^p d \ln p = 0.$$

This leads to

$$\frac{c_p}{R} \ln \frac{T}{T_o} = \ln \frac{p}{p_o}.$$

Taking antilog on both sides, we have

$$\left(\frac{T}{T_o} \right)^{c_p/R} = \frac{p}{p_o}. \quad (3.5.6)$$

- Eq. (3.5.6) is called the **Poisson's equation**, which is a very important equation for adiabatic processes. It expresses the temperature change as pressure changes in an adiabatic process.

If the temperature at the original pressure level p is T , then the final temperature after this adiabatic process will become

$$\theta = T \left(\frac{p_o}{p} \right)^{R/c_p} = T \left(\frac{p_o}{p} \right)^\kappa \quad (3.5.7)$$

where $\kappa=R/c_p$ is a constant, p in units of hPa (mb), and the would-be temperature θ is called **potential temperature**. For the dry air, $\kappa=287/1004=0.286$.

- As can be seen from Eqs. (3.5.6) and (3.5.7), θ is a constant (i.e. T_o at $p=p_o$) for an insulated gaseous system of fixed composition, i.e. for an adiabatic process. Thus, if an air parcel is brought adiabatically to a higher level (smaller p), then the temperature of this air parcel will drop due to expansion. On the other hand, the temperature will increase when the air parcel is brought down (p larger) due to compression.
- Eq. (3.5.6), i.e. the Poisson's equation, can be expressed in different forms, such as:

$$\begin{aligned} Tp^{-\kappa} &= \text{constant}, \\ p\alpha^\gamma &= \text{constant}, \\ T\alpha^{\gamma-1} &= \text{constant}, \end{aligned} \tag{3.5.8}$$

where $\kappa=R/c_p$ and $\gamma=c_p/c_v$ are constants.

- **Claim:** θ is a conservative property for adiabatic processes, i.e., $d\theta=0$ if $dq=0$.

Proof:

Starting from the definition (3.5.7),

$$\ln \theta = \ln T + \kappa(\ln p_o - \ln p)$$

Taking the derivative on the above equation,

$$d \ln \theta = d \ln T - \kappa \ln p \quad (3.5.9)$$

which leads to

$$\frac{d\theta}{\theta} = \frac{dT}{T} - \frac{R}{c_p} \frac{dp}{p},$$

or

$$(c_p T) \frac{d\theta}{\theta} = c_p dT - \alpha dp, \quad (3.5.10)$$

Comparing (3.5.10) with the 1st law

$$dq = c_p dT - \alpha dp,$$

we obtain

$$c_p T \frac{d\theta}{\theta} = dq = 0,$$

Since both T and θ cannot be zero, therefore $d\theta$ must be zero, i.e., **potential temperature is conserved for adiabatic processes.**

Eq. (3.5.9) may also be used to evaluate the change in potential temperature during a non-adiabatic or diabatic process.

➤ In the real atmosphere, many processes can contribute to heating, such as

- (a) surface radiative heating or cooling
- (b) latent heat due to phase change (condensation, evaporation, freezing, sublimation, etc.)
- (c) friction
- (d) transfer of sensible heat by turbulence (e.g., from the ocean), etc.

So, precisely speaking, many real atmospheric processes are not adiabatic.

Let's examine the above diabatic heating processes.

- (a) is important in the lower atmosphere since the most of the radiative energy the atmosphere receives is from the earth (long wave radiation). Radiation could be important on top of a stratus cloud, too;
- (b) latent heat is important for moist air and in regions of strong vertical motion (convection);
- (c) friction is important near the surface in the planetary boundary layer (PBL);
- (d) is important also near the surface.

So, most of the diabatic heat processes occur either near the surface or in/near the convective region. Thus, except in those regions, the atmosphere can be treated as approximately adiabatic.

The adiabatic process can also be represented in a α - p diagram. Before we do this, let us consider an isothermal

process, which can be represented by a curve of $p\alpha=RT=\text{constant}$.

Now, let draw a few of these isotherms and consider an adiabatic process by compressing a cylinder filled with air from volume α_2 to α_1 . Due to the compression, the temperature of the air in the cylinder during this adiabatic process will increase, so the adiabatic curve will be steeper than the isotherms.

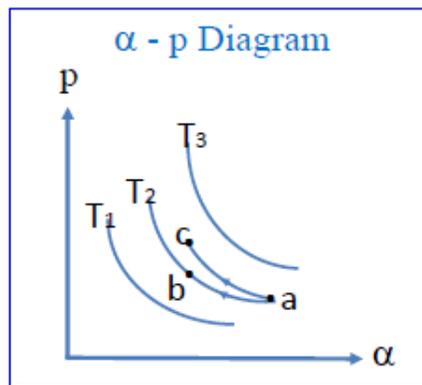


Fig.10.1: α - p diagram. $T_1 < T_2 < T_3$ are isotherms.

$a \rightarrow b$: isothermal process ($p\alpha=RT=\text{constant}$)

$a \rightarrow c$: adiabatic process

$$[p\alpha^{c_p/c_v} = p\alpha^{1.4} = \text{constant (steeper)}]$$