ASME 231 **Atmospheric Thermodynamics**
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Lecture 18 Saturated-adiabatic and pseudoadiabatic processes

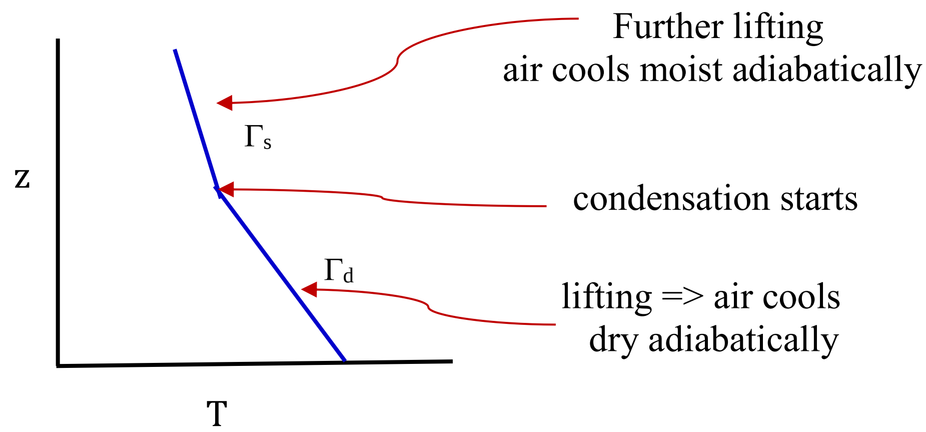
(Sec.4.6 of Hess)

[For classical equation editor: ($dq = 0$)]

- Concept of an air parcel: We assume that (e.g., Lin 2007, p.238)

  (a) The pressure of the air parcel adjusts immediately to the pressure of its environment ($\bar{p}$), i.e. $p = \bar{p}$, when it moves away from its initial level,
  (b) The environment of the air parcel is in hydrostatic balance,
  (c) No compensating motions exist in the parcel's environment, and
  (d) The air parcel does not mix with its environment and so retains its original identity.

➢ Lifting of a moist air parcel (Figure 19.1):



Dry adiabatic lapse rate: $\Gamma_d = -dT/dz|_{dry\ air}$

Saturated adiabatic lapse rate: $\Gamma_s = -dT/dz|_{saturated\ air}$

Two extreme cases of moist air processes

(1) Saturated-adiabatic process: All the condensates (rain, snow, and/or hail) remain in the rising air parcel, the process is still considered adiabatic (reversible), even though latent heat is released in the system, provided that heat does not pass through the boundaries of the parcel.

(2) Pseudoadiabatic process: All the condensates immediately fall out of the air parcel once they form. The process is irreversible and is not strictly adiabatic.

➢ Since the amount of heat carried by the condensation products is very small compared to that carried by the air parcel, so these two processes are almost identical (you may find out that in meteorology, many theories are in approximate form, instead of in exact form. They are almost no exact laws like in mathematics.)

Processes in the real atmosphere is somewhere in between the above two processes. Some liquid water and ice crystals fall out of the cloud as raindrops and snow or graupel/hail, while some are suspended as cloud droplets and cloud ice.

A sketch of a supercell thunderstorm and an example of numerical simulations of hailstorms:

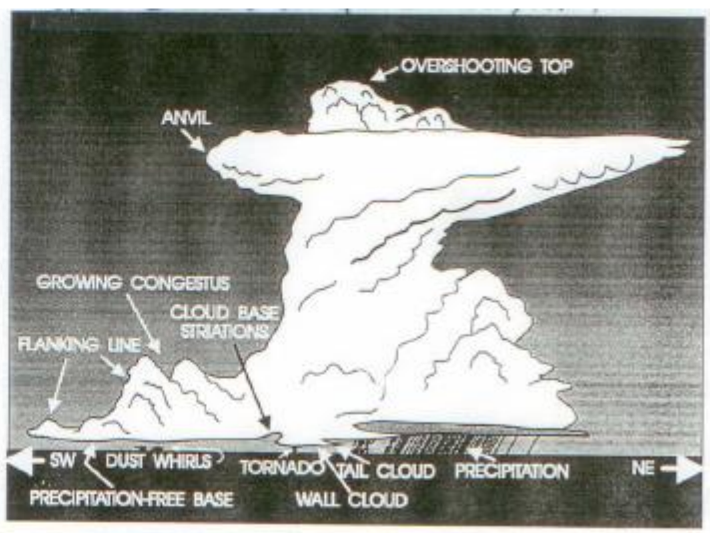


Fig. 19.2 Sketch of tornadic thunderstorm (Emanuel 1994)

Example: Suppose 5 g of water condense out of 1 kg of air during a moist-adiabatic expansion. For a further cooling of ΔT,

$$\frac{\text{Total internal energy associated with water}}{\text{Total internal energy associated with air parcel}}$$

$$= \frac{m_w c_w \Delta T}{m_a c_v \Delta T} = \frac{5x10^{-3} x4218}{1x717} = 0.029 << 1$$

Thus, the saturated-adiabatic lapse rate in the real atmosphere is almost the same as the pseudoadiabatic lapse rate.

- The saturated-adiabatic lapse rate: (Γ_s)

Let us consider the second form of the first law of thermodynamics,

$$dq = c_p dT - \alpha dp.$$

Since our goal is to find the temperature change with height, i.e. a relation between T and z, we need to do two things.

First, we would like to find the relation between α, p and z. This can be accomplished by using the hydrostatic equation, which is equivalent to the measure of pressure by the weight of the air column of 1 m^2 above it:

$$dp = -\rho g dz. \qquad (19.1)$$

The above equation is pretty accurate for most atmospheric applications, but not for strong convective processes.

Combine the 1st law and the hydrostatic equation, we have

$$dq = c_p dT + g dz \tag{19.2}$$

Next, we need to replace q with some variables usually measured in the atmosphere. If we know how much moisture is condensed, then we know the amount of latent heat released. Assume the mixing ratio is w_s, then

$$dq = -L dw_s \,. \tag{19.3}$$

For condensation, $dw_s < 0$, so that $dq > 0$. This means that the moisture contained in the moist air parcel is reduced, while an amount of latent heat is added to the moist air parcel.

Substituting (19.3) into Eq. (19.2) leads to

$$L dw_s = c_p dT + g dz \,. \tag{19.4}$$

Divide (19.4) by Δz and taking the limit $\Delta z \to 0$ give us

$$\frac{dT}{dz} = -\frac{L}{c_p}\frac{dw_s}{dz} - \frac{g}{c_p}, \tag{19.5}$$

or

$$\frac{dT}{dz} = -\frac{L}{c_p}\frac{dw_s}{dT}\frac{dT}{dz} - \frac{g}{c_p}. \tag{19.6}$$

The above equation may be rearranged to be

$$\frac{dT}{dz}\left(1 + \frac{L}{c_p}\frac{dw_s}{dT}\right) = -\frac{g}{c_p}.$$

Therefore, we may obtain the moist lapse rate,

$$\Gamma_s = -\frac{dT}{dz} = \frac{g/c_p}{1 + \frac{L}{c_p}\frac{dw_s}{dT}}. \tag{19.7}$$

The dry lapse rate, Γ_d, may be reduced by letting dq=0 in (19.2) to obtain

$$\Gamma_d = -\frac{dT}{dz} = \frac{g}{c_p}. \tag{19.8}$$

Discussions:

(i) Γ_d is a constant;

(ii) Γ_s is not a constant. $\Gamma_s = \Gamma_d$ if there exists no moisture.

(iii) $\Gamma_s < \Gamma_d$ since $dw_s/dT > 0$ (at higher temperature, more moisture is needed to saturate the air). Physically it is due to the latent heat added to the air parcel.

(iv) Observations indicate that is

- approximately equal to $4\ ^oC/km$ near ground in warm humid air;
- $6-7\ ^oC/km$ in middle troposphere;
- only slightly less than Γ_d near the tropopause.

(v) It is difficult to measure dw_s/dT, but can be calculated explicitly since $e_s(T)$ is known. Notice that

$$w_s = \varepsilon \frac{e_s}{p}, \ \ \varepsilon = 0.622$$

- Using the above equation and the Clausius-Clapeyron equation, we can obtain

$$\frac{dw_s}{dT} = \frac{Lw_s}{R_v T^2} + \frac{gw_s}{R_d T\, \Gamma_s}$$

We can substitute the above equation into (19.7) to obtain the saturated adiabatic lapse rate,

$$\Gamma_s = \frac{\Gamma_d (1 + Lw_s / R_d T)}{1 + L^2 w_s / c_p R_v T^2}.$$