

## Lecture 19 Moisture Variables

(Sec.4.7 of Hess)

[For classical equation editor: ( $dq = 0$ )]

### a. Vapor pressure ( $e$ )

When water vapor is mixed with air, the pressure exerted by the  $H_2O$  is called the vapor pressure ( $e$ ), as mentioned before. The amount of vapor that can remain gaseous in an air parcel is limited, which gives the saturation vapor pressure ( $e_s$ ).

The saturation vapor pressure is a function of temperature only. This dependence upon temperature may be calculated from the Clausius-Clapeyron equation.

$$\frac{de_s}{dT} = \frac{L_{12}}{T(\alpha_2 - \alpha_1)}. \quad (18.5)$$

### b. Absolute humidity ( $\rho_v$ )

The absolute humidity is simply the density of water vapor, measured in  $kg\ m^{-3}$ .

### c. Mixing ratio ( $w$ )

As defined earlier, the mixing ratio is the ratio of the mass of water vapor present to the mass of dry air containing the vapor.

$$w = \frac{m_v}{m_d} = \frac{\rho_v}{\rho_d} = \varepsilon \frac{e}{p - e} \approx \varepsilon \frac{e}{p} \approx 0.622 \frac{e}{p}.$$

The saturation mixing ratio is given by

$$w_s = \varepsilon \frac{e_s}{p}.$$

Because  $w_s$  is a function of two thermodynamic variables, lines of constant value of  $w_s$  can be plotted on an  $\alpha$ - $p$  diagram or any thermodynamic diagrams.

Mixing ratio is expressed in practice as the number of gram per kilogram ( $g \text{ kg}^{-1}$ ) of dry air.

#### **d. Specific humidity ( $q$ )**

The specific humidity is defined as the ratio of the mass of water vapor to the mass of moist air containing the vapor. Thus,

$$q = \frac{m_v}{m_v + m_d} = \frac{\rho_v}{\rho} = \varepsilon \frac{e}{p}.$$

As can be seen, **specific humidity and mixing ratio are nearly equal** and have similar properties.

#### **e. Relative humidity ( $RH$ or $r$ )**

The relative humidity is **the ratio of the actual mixing ratio of an air parcel at a given temperature and pressure to the saturation mixing ratio of the air at that temperature and pressure:**

$$RH = r = \frac{w}{w_s}.$$

Thus, relative humidity simply gives the fraction of the saturation limit of vapor content which is actually possessed by a given air parcel (i.e.  $RH = r = e/e_s$ ).

It is usually multiplied by 100 and expressed in per cent, and is a function of  $w$ ,  $p$ , and  $T$ . In addition, since the saturation vapor pressures over ice and water differ, it is necessary to specify which is being used at temperature below 0°C.

#### **f. Virtual temperature ( $T_v$ )**

As defined before, the virtual temperature is the temperature at which dry air would have to be in order to have the same density as a moist air parcel, assuming both have the same pressure.

Thus, when the virtual temperature is used instead of the actual temperature the equation of state for dry air may be used for moist air parcel also.  $T_v$  may be calculated by

$$T_v = \frac{1 + w/\epsilon}{1 + w} \approx (1 + 0.61w)T.$$

Thus,  $T_v$  is a function of  $T$  and  $w$ .

#### **g. Dew-point temperature ( $T_d$ )**

The dew point temperature is defined as the temperature to which moist air must be cooled during a process in which  $p$  and

$w$  remain constant, in order that it shall become just saturated with respect to water.

Obviously at  $T_d$  the mixing ratio of the air becomes its saturation mixing ratio. Thus the mixing ratio lines on any thermodynamic diagram can be used to determine dew point graphically.

A similar definition can be given for the *frost-point temperature*, the only difference being that saturation with respect to ice must be achieved.

A meteorological process by which air can be brought to its dew point is radiative cooling of a layer of air near the ground during the night. Indeed, this is the primary process by which dew forms.

### **h. The lifting condensation level (*LCL*)**

*LCL* is the level to which unsaturated air would have to be raised in a dry expansion to produce condensation. At the *LCL* the mixing ratio becomes equal to the saturation mixing ratio. This level is found most easily by graphical means on a thermodynamic diagram. It is the level at which the dry adiabat through the mixing ratio line whose value of  $w_s$  is equal to the actual initial mixing ratio of the air parcel.

### **i. The wet-bulb temperature ( $T_w$ )**

$T_w$  may be defined in two ways which differ but slightly numerically.

The wet-bulb temperature may be defined to be the temperature to which air may be cooled by evaporating water into it at constant pressure until it is saturated. Note that  $w$  is not kept constant in this definition, so the wet-bulb temperature is different, in general, from the dew-point temperature.

When a *psychrometer* is used to measure moisture, the air is caused to move rapidly past two thermometer bulbs. One of these is dry and indicates the air temperature. The other is covered with a moist cloth and comes to thermal equilibrium at a temperature below that of the air, because the wet bulb is cooled by evaporation of some of its moisture. The calculation of  $T_w$  may be performed by numerical approximation or graphically. If  $T$  and  $T_w$  are measured with a psychrometer,  $w$  can be found directly.

It is immediately apparent that  $T_w$  lies between  $T$  and  $T_d$ . First,  $T_w \leq T$  (unless the air is already saturated). Second,  $T_w$  is the saturation temperature for  $w'$  (original  $w$  plus the additional vapor) while  $T_d$  is the saturation temperature for  $w$ . Since  $w < w'$ , we conclude that  $T_d \leq T_w$ . Thus, we have  $T_d \leq T_w \leq T$ .

The wet-bulb temperature may be obtained approximately via an adiabatic process and is best described as in the attached figure.

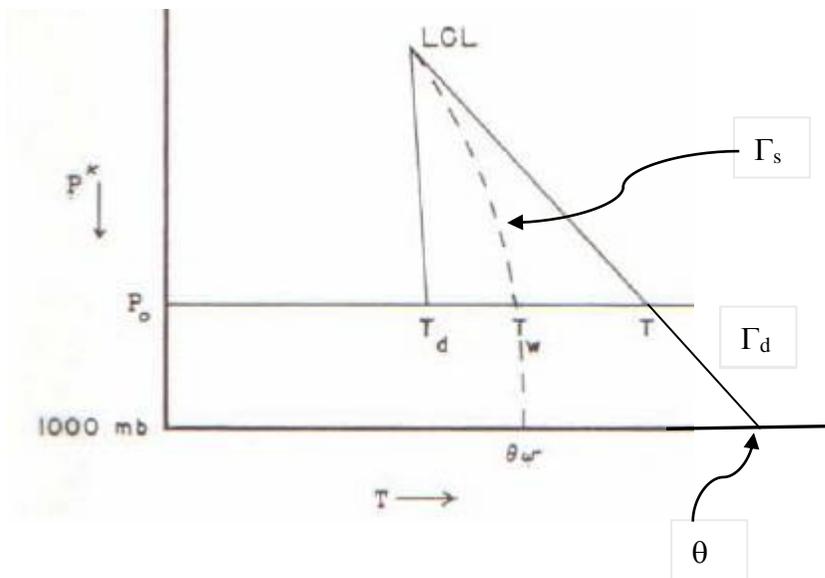


Figure 20.1 (Fig. 4.7 of Hess)

### j. The wet-bulb potential temperature ( $\theta_w$ )

The wet-bulb potential temperature is most easily defined in terms of graphical operations on a thermodynamic diagram (see Fig. 20.1):

- (1) Consider the pseudoadiabat which passes through the wet-bulb temperature, and
- (2) Follow this adiabat until it intersects the 1000 mb isobar. The temperature at this intersection is  $\theta_w$ .

### k. Equivalent temperature ( $T_e$ )

$T_e$  is the temperature an air parcel would have if all its latent heat were converted to sensible heat by means of a pseudoadiabatic expansion to low pressure and temperature followed by a dry adiabatic compression to the original pressure. Mathematically,

$$T_e = T e^{Lw_s / c_p T}$$

Graphically,  $T_e$  can be determined on a thermodynamic diagram as shown in the following figure.

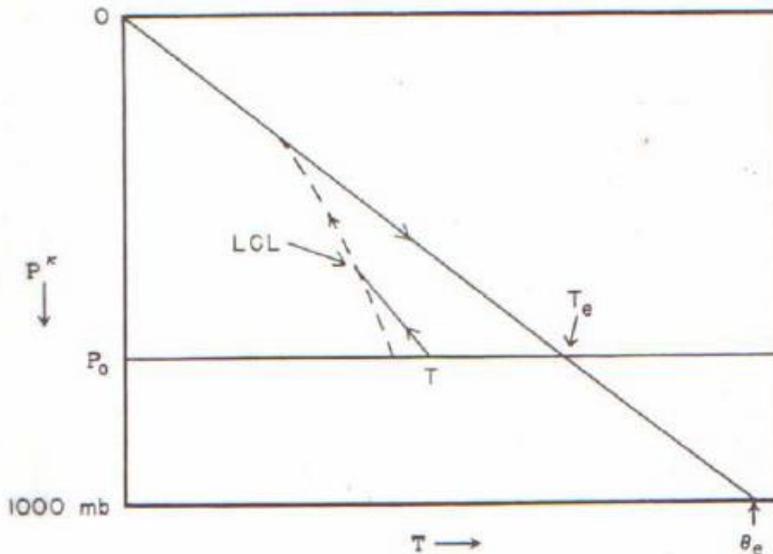


Figure 20.2 (Fig. 4.8 of Hess)

## I. Equivalent potential temperature ( $\theta_e$ )

The equivalent potential temperature is an air parcel would have if it were taken from its equivalent temperature to a pressure of 1000 mb via a dry adiabatic process. Mathematically,

$$\theta_e = \theta e^{Lw_s / c_p T}$$

[Reading Assignment]

[Derivation of Equivalent potential temperature \( \$\theta\_e\$ \)](#)

The concept of equivalent potential temperature can be considered as the potential temperature of moist air.

We derived the expression for potential temperature

$$\theta = T \left( \frac{p_o}{p} \right)^{R/c_p} . \quad (20.1)$$

using the 1st law of thermodynamics by setting  $q=0$ .

We have also shown, starting from (20.1),

$$ds = \frac{dq}{T} = \frac{c_p d\theta}{\theta} . \quad (20.2)$$

For moist air,

$$dq = -Ldw_s .$$

Thus,

$$-\frac{L}{c_p T} dw_s = \frac{d\theta}{\theta} . \quad (20.3)$$

Since (we will show in the next Lab),

$$\frac{L}{c_p T} dw_s \approx d \left( \frac{Lw_s}{c_p T} \right) . \quad (20.4)$$

Thus,

$$-\frac{L}{c_p} d\left(\frac{w_s}{T}\right) = d(\ln\theta).$$

Integrate the above equation for the following process (Figure 20.2).

Where the initial state is  $p=1000$  mb, mixing ratio is  $w_s$  and potential temperature is  $\theta$ , the final state is a state of dry air (all moisture has been condensed out,  $w_s = 0$ ). Denote the potential temperature at the final state as  $\theta_e$ , because condensation occurred in the process,  $\theta_e$  will not be  $\theta$ .

$$\int_{w_s/T}^0 -\frac{L}{c_p} d\left(\frac{w_s}{T}\right) = \int_{\theta}^{\theta_e} d(\ln\theta).$$

Thus,

$$\frac{Lw_s}{c_p T} = \ln\left(\frac{\theta_e}{\theta}\right)$$

or

$$\theta_e = \theta e^{Lw_s/c_p T}. \tag{20.5}$$

This is called the "equivalent potential temperature".

Discussions:

- i) If  $w_s = 0$  (no moisture),  $\theta_e = \theta$ ;

- ii)  $\theta_e$  can be found in the following way: air parcel rises and expands pseudoadiabatically (i.e., along  $\Gamma_d$  to LCL and then along  $\Gamma_s$  until all moisture has condensed and latent heat released, and all liquid water droplets fallen out). The equivalent potential temperature  $\theta_e$  is potential temperature  $\theta$  at that point which can be easily found by following the dry adiabat  $\Gamma_d$  starting from that point and descending to 1000 mb. Then the temperature there (at 1000 mb) is  $\theta_e$ ;
- iii)  $\theta_e$  is conserved during both dry and saturated adiabatic processes, i.e,  $\theta_e$  is constant during pseudoadiabatic processes.

### Derivation of equivalent temperature ( $T_e$ )

In the process depicted in Figure (20.1), if the air parcel descends to the original pressure level rather than 1000 mb after it reached the level of  $w_s = 0$ , then the new temperature at the original pressure level ( $p$ ) is called the equivalent temperature ( $T_e$ ).

How do we express  $T_e$ ?

$T_e$  is related to  $\theta_e$  by the Poisson's equation.

$$T_e/\theta_e=(p/1000)^\kappa$$

But

$$T/\theta = (p/1000)^\kappa$$

so

$$T/\theta = T_e/\theta_e$$

or

$$T_e = T(\theta_e/\theta)$$

From (20.5),

$$T_e = T e^{LW_s/(c_p T)}$$

Discussion:

- (i) The relation between  $T_e$  and  $T$  is the same as that between  $\theta_e$  and  $\theta$ ;
- (ii)  $T_e > T$  as can be seen from (20.7), since air at its equivalent temperature has realized all its latent heat;
- (iii) In the above theory,  $T$  and  $w_s$  are the values in any state of a saturated adiabatic process. If the air parcel is initially unsaturated, then  $w_s$  and  $T$  are values at the point of initial saturation ( $LCL$ ). But,  $w_s$  at  $LCL$  is the same as the initial value since before condensation occurs,  $w_s$  is conserved.