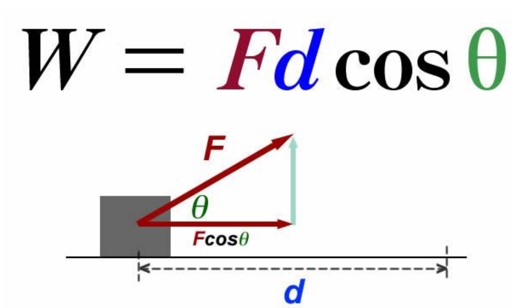
Lecture 5

(Chap. 3: First Law of Thermodynamics)
(classical equation editor: cut & paste $\Delta Q = C_2(T_F - T_2)$)

3.1 Work

Definition of work: The amount of work W done by a force F which displaces a mass over a distance d in the direction of the force is $Fd \cos \theta$. The symbol θ denotes the angle between the direction of actual displacement and the force.

Figure 6.1:



Thus the amount of work dW done by a force F which displaces a mass over a small distance dS is

$$dW = FdS = F\, dS \cos \theta. \quad (3.1)$$

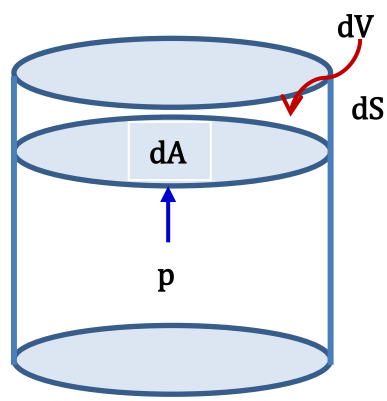
Work of expansion is defined as the work done by a force in gas expansion. The work of expansion is often represented by pressure. Since pressure is defined as the force per unit area ($p = F/dA$), the force of expansion may be written as $F = p\, dA$.

Thus, the work of expansion can be expressed as

$$dW = Fds = p\, dA\, ds = pdV, \tag{3.2}$$

where dV is the volume element given by the cylinder swept by dA in the direction of F.

Figure 6.2:



Divide Eq. (3.2) by the mass of the fluid system m, we obtain

$$dw = p d\alpha, \qquad (3.3)$$

where $d\alpha$ is the specific volume and $dw=dW/m$ is called the specific work and the sign of dw is defined as follows:

$dw > 0$ if the system expands and does work on its environment;
$dw < 0$ if the system is compressed by an external pressure force.

For a finite expansion,

$$w = \int_i^f p d\alpha \qquad (3.4)$$

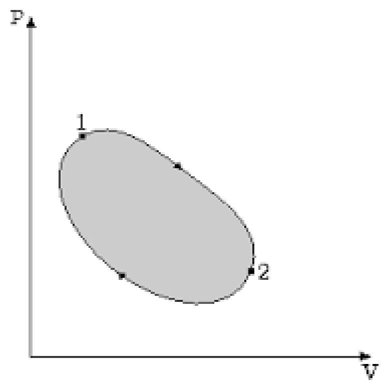
where subscripts i and f stand for the initial and final states, respectively.

For a cyclic process as shown in the figure (in α-p space),

$$w = \oint p d\alpha = \int_1^2 p d\alpha + \int_2^1 p d\alpha$$

Mathematically, w is the area enclosed by the curve and $w > 0$ if the curve is clockwise.

Figure 6.3:



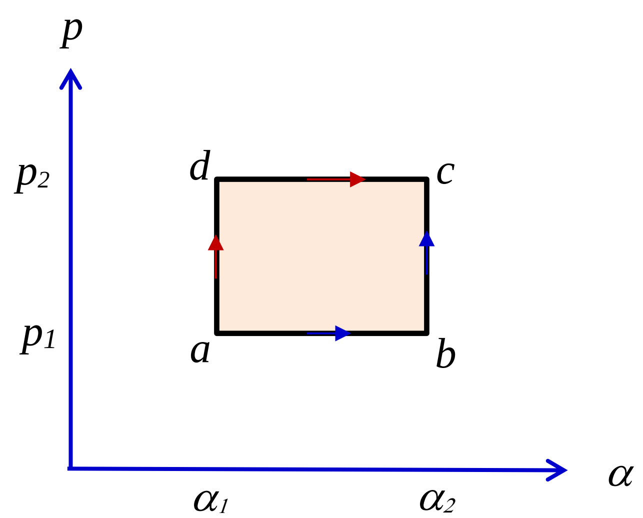
The work of expansion is the only work we shall consider in our atmospheric system.

In an isobaric process, p is a constant. Thus, the total work done can be computed from Eq. (3.4).

In an isothermal process, p is only a function of α. Thus, the total work done can also be computed from Eq. (3.4).

In an isosteric process, there is no change in volume (no expansion or $d\alpha = 0$). Thus, the work done is zero.

Example: Consider the following processes (Fig. 6.4)



Path A ($a \rightarrow b \rightarrow c$): The system goes through an isobaric expansion from *a* to *b* (heating the cylinder while keeping the pressure constant), and then goes through an isosteric pressure increase (such as heating the cylinder while fix the piston) from *b* to *c*.

The work done is

$$w_A = p_1(\alpha_2 - \alpha_1)$$

Path B ($a \rightarrow d \rightarrow c$): The system goes through an isosteric pressure increase (e.g. heating the cylinder while fix the piston) from a to d followed by an isobaric expansion fro*m d to c (e.g. heating the cylinder while keeping the pressure constant).*

The work done is

$$w_B = p_2(\alpha_2 - \alpha_1)$$

Thus, the work done depends on the process (i.e., path on a α-p diagram).

Path C: For a cyclic process ($a \to b \to c \to d \to a$), the work done can be calculated

$$w_C = p_1(\alpha_2 - \alpha_1) + 0 + p_2(\alpha_1 - \alpha_2) + 0 = (p_1 - p_2)(\alpha_2 - \alpha_1) \neq 0$$

Thus, work done may not be zero for a cyclic process.

In summary,

(1) w depends on the path of the process ($w_1 \neq w_2$).

(2) w may be non-zero even for cyclic processes.

(1/31/17)